21.1 Investigating the Derivative of sin x Graphically, Numerically, and Using Physical Intuition
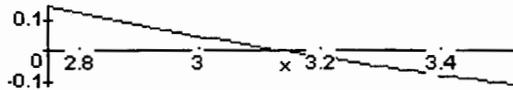
1. $\frac{\sin 3}{3} = 0.0470$, $\frac{\sin 3.14}{3.14} = 0.0005$,

2. $f'(\pi) \cong \frac{\sin(\pi + 0.0001) - \sin(\pi)}{0.0001} \cong -1.000$

3. (a)

x	1.998	1.999	2.000	2.001	2.002
$f(x)$	0.910128	0.909713	0.909297	0.908881	0.908463

(b) $f'(1.999) \cong \frac{f(1.998) - f(1.999)}{1.998 - 1.999} \cong -0.415$, or $\frac{f(2) - f(1.999)}{2 - 1.999} \cong -0.416$, average is -0.4155

$f'(2) \cong \frac{f(1.999) - f(2)}{1.999 - 2} \cong -0.416$, or $\frac{f(2.001) - f(2)}{2.001 - 2} \cong -0.416$, they agree at -0.416

$f'(2.001) \cong \frac{f(2) - f(2.001)}{2 - 2.001} \cong -0.416$, or $\frac{f(2.002) - f(2.001)}{2.002 - 2.001} \cong -0.418$, average is -0.417

(c) $f''(2) \cong \frac{f'(1.999) - f'(2)}{1.999 - 2} \cong -0.5$, or $\frac{f'(2.001) - f'(2)}{2.001 - 2} \cong -1$, so averaging $f''(2) \approx -0.75$

Makes one curious what the results would be if used more digits of accuracy in the calculations.

21.2

3. (a) $y = 5\cos x$ $\qquad$ $y' = -5\sin x$ $\qquad\qquad\qquad$ $y'' = -5\cos x$

(b) $y = -3\sin(2x)$ $\qquad$ $y' = -3\cos(2x)2 = -6\cos(2x)$ $\qquad$ $y'' = 6\sin(2x)2 = 12\sin(2x)$

(c) $y = .5\tan x$ $\qquad$ $y' = .5\sec^2 x$ $\qquad\qquad\qquad$ $y'' = .5(2\sec x(\sec x \tan x)) = \sec^2 x \tan x$

(d) $y = 2\sin x \cos x$; $\quad$ $y' = 2(\cos^2 x - \sin^2 x)$; $\quad$ $y'' = 2(2\cos x(-\sin x) - 2\sin x \cos x) = -8\sin x \cos x$
$\qquad$ OR equivalently $y = \sin(2x)$; $\qquad$ $y' = 2\cos(2x)$; $\qquad$ $y'' = -4\sin(2x)$

6. (a) $\frac{d}{dx}\sin(u(x)) = \cos(u(x))\frac{d}{dx}(u(x)) = \cos(u(x))u'(x)$

(b) $\frac{d}{dx}\cos(u(x)) = -\sin(u(x))\frac{d}{dx}(u(x)) = -\sin(u(x))u'(x)$

(c) $\frac{d}{dx}u(x)\sin x = u'(x)\sin x + u(x)\cos x$

8. $\frac{d}{dx}\sin(x^3 + \ln 3x) = \cos(x^3 + \ln 3x)[3x^2 + \frac{1}{3x}(3)] = \cos(x^3 + \ln 3x)(3x^2 + \frac{1}{x})$

10. $\frac{d}{dx}[\frac{1}{\sin^3(\cos 2x)}] = \frac{d}{dx}[\sin(\cos 2x)]^{-3} = -3[\sin(\cos 2x)]^{-4}(\cos(\cos 2x)(-\sin 2x)(2)) = \frac{6\sin 2x \cos(\cos 2x)}{\sin^4(\cos 2x)}$

12. $\frac{d}{dx}\left(4[2 - \cos(\frac{x}{7})]^{-\frac{1}{2}}\right) = 4(-\frac{1}{2})[2 - \cos(\frac{x}{7})]^{-\frac{3}{2}}\sin(\frac{x}{7})(\frac{1}{7}) = \frac{-2\sin(\frac{x}{7})}{7(2 - \cos(\frac{x}{7}))^{\frac{3}{2}}}$

14. $\sin(xy) + y = y\cos x \Rightarrow \cos(xy)[(1)y + x\frac{dy}{dx}] + \frac{dy}{dx} = \frac{dy}{dx}\cos x + y(-\sin x)$

$\Rightarrow y\cos(xy) + x\cos(xy)\frac{dy}{dx} + \frac{dy}{dx} = \frac{dy}{dx}\cos x - y\sin x$

$\Rightarrow \frac{dy}{dx}(x\cos(xy) - \cos x + 1) = -y\cos(xy) - y\sin x \Rightarrow \frac{dy}{dx} = \frac{y\cos(xy) + y\sin x}{\cos x - x\cos(xy) - 1}$