21.3 Applications

1. (a) $f'(x) = 1 + 2\cos x = 0 \Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow x = \frac{2}{3}\pi + k(2\pi)$ *or* $x = \frac{4}{3}\pi + k(2\pi)$

(b)

+	0	-	0	+	0	-	0	+
	$\frac{2}{3}\pi$		$\frac{4}{3}\pi$		$\frac{8}{3}\pi$		$\frac{10}{3}\pi$	

decreasing on $[\frac{2}{3}\pi + 2k\pi, \frac{4}{3}\pi + 2k\pi]$

increasing on $[-\frac{2}{3}\pi + 2k\pi, \frac{2}{3}\pi + 2k\pi]$

(c) local maxima at $\frac{2}{3}\pi + 2k\pi$ and local minima at $\frac{4}{3}\pi + 2k\pi$

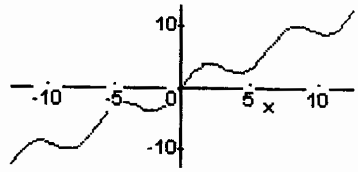
(d) no global max or min as keeps on climbing

(e) $f''(x) = -2\sin x = 0$ for $x = k\pi$

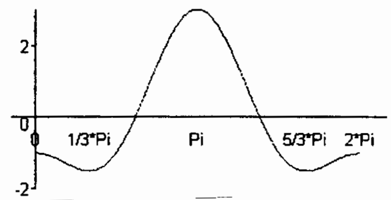
-	0	+	0	-	0	+	0	-
	π		2π		3π		4π	

therefore f is concave up on $[(2n-1)\pi, 2n\pi]$ and concave down on $[2n\pi, (2n+1)\pi]$

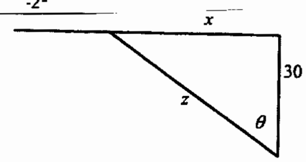
f)



5. $f'(x) = -2\sin 2x + 2\sin x = -4\sin x\cos x + 2\sin x$
$= 2\sin x(-2\cos x + 1) = 0 \Leftrightarrow \sin x = 0$ *or* $\cos x = \frac{1}{2}$
$\Rightarrow x = 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$



8. $\tan\theta = \frac{x}{30} \Rightarrow \sec^2\theta \frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt} \Rightarrow$
$\frac{d\theta}{dt} = \cos^2\theta \frac{1}{30}\frac{dx}{dt} \Rightarrow \frac{d\theta}{dt}\Big|_{x=50} = \frac{900}{3400}\frac{1}{30\,ft}(-46\frac{ft}{sec}) \approx -0.4059\frac{rad}{sec}$



(when $x = 50$, $z^2 = 900 + 2500 = 3400$ so $\cos^2\theta = \frac{900}{3400}$)

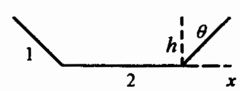
12.(a) $x(\theta) = \frac{v_0^2}{g}\sin(2\theta) \Rightarrow x'(\theta) = \frac{v_0^2}{g}\cos(2\theta)2 = 0 \Leftrightarrow \cos(2\theta) = 0 \Leftrightarrow \theta = \frac{\pi}{4}$

$x_{max} = \frac{v_0^2}{g}\sin(2(\frac{\pi}{4})) = \frac{v_0^2}{g}$

(b) $\frac{100mi}{hr} = \frac{100mi}{hr}\frac{5280\,ft}{mi}\frac{100m}{328\,ft}\frac{1hr}{3600\sec} \cong \frac{44.7m}{\sec} \Rightarrow x_{max} \cong \frac{(44.7\,m/_{sec})^2}{9.8\,m/_{sec^2}} \cong 204m$.

($\cong 669\,ft$ which is not very realistic as wind resistance is not taken into account.)

14. $V = 8(2+x)h = 8(2+\sin\theta)\cos\theta$



$V' = 8[\cos\theta(\cos\theta) + (2+\sin\theta)(-\sin\theta)] = 8[-2\sin^2\theta - 2\sin\theta + 1]$

$V' = 0 \Leftrightarrow \sin\theta = \frac{2\pm\sqrt{12}}{-4} = \frac{1\pm\sqrt{3}}{-2} \Rightarrow \theta \cong 0.3747\,rad \cong 21.47^\circ$ (negative sin is unrealistic)

23. $z^2 = x^2 + y^2 \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ with $\frac{dx}{dt} = -5\frac{ft}{sec}$ and $\frac{dy}{dt} = 10\frac{ft}{sec}$

(a) When $x = 200$ and $y = 100$, $z = 100\sqrt{5}$ hence $100\sqrt{5}\frac{dz}{dt} = 200(-5) + 100(10) = 0$, thus the distance between them is neither increasing or decreasing.

(b) $\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2} \Rightarrow \frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}\cos^2\theta$.

At that instant $\cos\theta = \frac{200}{100\sqrt{5}}$, thus

$$\frac{d\theta}{dt} = \frac{200(10) - 100(-5)}{(200)^2}\left(\frac{2}{\sqrt{5}}\right)^2 = \frac{2500}{40000}\frac{4}{5} = \frac{5}{100}\frac{rad}{sec}$$