Math Xb Spring 2004
Worksheet: Derivatives of Inverse Trig Functions
March 19, 2004

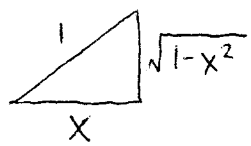
1. Let $f(x) = \cos^{-1} x$.

   (a) Prove that $f'(x) = -\frac{1}{\sqrt{1-x^2}}$.

$y = \cos^{-1}(x)$

$\cos(y) = x$

$-\sin y \frac{dy}{dx} = 1$



$$\frac{dy}{dx} = -\frac{1}{\sin y} = \frac{-1}{\sin(\cos^{-1}(x))} = \frac{-1}{\sqrt{1-x^2}}$$

   (b) Use the formula for $f'(x)$ to explain why f is a decreasing function on its entire domain, $[-1, 1]$.

the denominator of $\frac{-1}{\sqrt{1-x^2}}$ is ≥ 0 for x in $[-1,1]$ and the numerator is < 0, so the expression is always $< 0 \Rightarrow$ f is decreasing

   (c) Find $f''(x)$ and use it to determine the intervals on which f is concave up and concave down.

$$f'(x) = -(1-x^2)^{-1/2} \Rightarrow f''(x) = \frac{1}{2}(1-x^2)^{-3/2} \cdot (-2x) = \frac{-2x}{2(1-x^2)^{3/2}} = \frac{-x}{2(1-x^2)^{3/2}}$$

concave down for $0 < x \leq 1$

concave up for $-1 \leq x < 0$

2. Find the derivative of each of the following functions.

   (a) $f(x) = (\sin^{-1} x)^2$

$$f'(x) = \frac{2(\sin^{-1} x)}{\sqrt{1-x^2}}$$

   (b) $f(x) = \sin^{-1}(x^2)$

$$f'(x) = \frac{2x}{\sqrt{1-x^4}}$$

   (c) $f(x) = (\tan^{-1} x) \ln x$ $= \frac{\tan^{-1} x}{x} + \frac{\ln x}{1+x^2}$

21.4 Derivatives of Inverse Trigonometric Functions

1. $y' = 3\sec^2 x - 4\frac{1}{1+x^2}$

2. $f'(x) = 3\frac{1}{1+(2\sqrt{x})^2}2\frac{1}{2}x^{-\frac{1}{2}} = \frac{3}{(1+4x)\sqrt{x}}$

3. $y' = \cos x \arcsin x + \sin x \frac{1}{\sqrt{1-x^2}}$

4. $y = (\tan^{-1} x)^{1/2} \Rightarrow y' = \frac{1}{2}(\tan^{-1} x)^{-1/2}\frac{1}{1+x^2} = \frac{1}{2(1+x^2)\sqrt{\tan^{-1} x}}$

5. $y' = (1)\tan^{-1} x + x\frac{1}{1+x^2} = \tan^{-1} x + \frac{x}{1+x^2}$