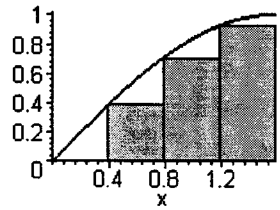
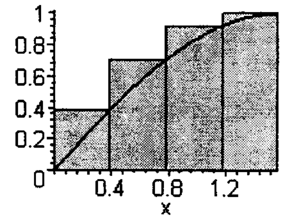
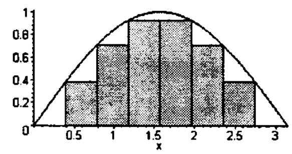
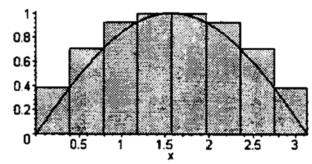
4. (a) lower bound = left sum ≈.79
upper bound = right sum ≈1.18





(b) lower bound ≈ 1.58
upper bound ≈ 2.36
(These are twice the numbers in part (a).)





(c) Same answers as part (b) as $\cos x$ on $[-\pi/2, \pi/2]$ is the same as $\sin x$ on $[0, \pi]$ shifted left.

1. A: $\int_0^5 f(t)\,dt$ greatest distance, B: $\int_0^5 g(t)\,dt$, C: $\int_0^5 h(t)\,dt$ least distance

4. (a) $L_4 = (0)^3 \frac{1}{2} + (\frac{1}{2})^3 \frac{1}{2} + (\frac{2}{2})^3 \frac{1}{2} + (\frac{3}{2})^3 \frac{1}{2} = \frac{9}{4}$

$R_4 = (\frac{1}{2})^3 \frac{1}{2} + (\frac{2}{2})^3 \frac{1}{2} + (\frac{3}{2})^3 \frac{1}{2} + (\frac{4}{2})^3 \frac{1}{2} = \frac{25}{4}$

(b) $L_6 = \left(\frac{1}{\frac{3}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{4}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{5}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{6}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{7}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{8}{3}}\right)\frac{1}{3} = \frac{341}{280} \approx 1.218$

$R_6 = \left(\frac{1}{\frac{4}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{5}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{6}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{7}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{8}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{9}{3}}\right)\frac{1}{3} = \frac{2509}{2520} \approx 0.9956$

5. Note: We assume that $v_g(t)$ is zero on [0, 1] and [7, 8].

(a) The area under $v_c(t)$ on [0, 1] which is $\int_0^1 v_c(t)dt$.

(b) The area under $v_c(t)$ minus area under $v_g(t)$ on [0, 1.5] is $\int_0^{1.5} v_c(t)dt - \int_0^{1.5} v_g(t)dt$.

(c) The absolute value of [the area under $v_c(t)$ minus area under $v_g(t)$] on [0, 3]
$= \left|\int_0^3 v_c(t)dt - \int_0^3 v_g(t)dt\right|$, (we don't know which quantity is largest).

(d) $t = 6.5$ $v_g(t)$ was $> v_c(t)$ so the goat kept getting further ahead. After $t = 6.5$ $v_g(t) < v_c(t)$.

(e) $t \approx 3$ Areas under velocity functions on [0, 3] are approximately equal.

(f) $t \approx 4.2$ The difference in velocities is the greatest at $t = 4.2$, $v_g'(4.2) = v_c'(4.2)$.

6. (a) $L_4 < L_{20} < L_{100} < \int_0^2 x^3 dx < R_{100} < R_{20} < R_4$.

(b) $|R_4 - L_4| = \left|2^3 \frac{2}{4} - 0^3 \frac{2}{4}\right| = 4 = \frac{2^4}{4}$.

(c) $|R_{100} - L_{100}| = \left|2^3 \frac{2}{100} - 0^3 \frac{2}{100}\right| = \frac{2^4}{100} = 0.16$

(d) $|R_n - L_n| = \left|2^3 \frac{2}{n} - 0^3 \frac{2}{n}\right| = \frac{2^4}{n} < 0.05 \Leftrightarrow \frac{2^4}{0.05} < n \Leftrightarrow 320 < n$

(e) $R_4 = \sum_{k=1}^{4} (\frac{k}{2})^3 \frac{1}{2} = (\frac{1}{2})^3 \frac{1}{2} + (\frac{2}{2})^3 \frac{1}{2} + (\frac{3}{2})^3 \frac{1}{2} + (\frac{4}{2})^3 \frac{1}{2} = \frac{25}{4}$