Chapter 22

Net Change in Amount and Area: Introducing the Definite Integral

22.1 Finding Net Change in Amount: Physical and Graphical Interplay

1. These results are approximate, dependent upon eyeball interpretation of the graph in the text.

(a) 10 AM as after 10 AM the rate of arrival is greater than the service rate.

(b) $\approx$ 12 noon. After this the arrival rate decreases.

(c) $\approx$ 2 PM. After this the arrival rate is less than the service rate.

(d) $\approx$ 165 – 120 = 45 people. Arrivals(area under $r(t)$ 10 to 2) minus those served 10 to 2.

(e) Those arriving at 2 PM must wait through the longest line. 45/15 = 3 hours to wait.

(f) $\approx$ 45 + 23 – 30 = 32. Those in line, plus arrivals minus those served.

(g) $\approx$ 225 people (area under curve.)

22.3 The Definite Integral: Qualitative Analysis and Signed Area

1. (a) Area under curve is triangle $\frac{1}{2}5(5) = \frac{25}{2}$
   (b) Area above – area below x-axis = 0
   (c) Area is $2[\frac{1}{2}(2)2] = 4$
   (d) Area is 4(3)=12
   (e) Area above – area below x-axis = 0
   (f) Area above – area below x-axis = 0
   (g) Above – below = $\frac{1}{2}(3)(3) - \frac{1}{2}(1)(1) = 4$
   (h) Area = $\frac{1}{2}(3)(3) + \frac{1}{2}(1)(1) = 5$

3. $b < c = e < d < a < f$

5. Signed area on [-8, -2] is 12, on [-2, 0] is 4, on [0, 3] is $-\frac{1}{4}\pi 3^2 = -\frac{9}{4}\pi$, and on [3, 6] is $-\frac{9}{4}\pi$
   (a) 12
   (b) 12+4=16
   (c) $-\frac{9}{4}\pi + (-\frac{9}{4}\pi) = -\frac{9}{2}\pi$
   (d) $-\frac{9}{4}\pi$

6. Signed area on [-8, -2] is 12, on [-2, 0] is 4, on [0, 3] is $-\frac{1}{4}\pi 3^2 = -\frac{9}{4}\pi$, and on [3, 6] is $-\frac{9}{4}\pi$
   (a) $4 - \frac{9}{4}\pi$
   (b) $12 + 4 + -\frac{9}{2}\pi = 16 - \frac{9}{2}\pi$
   (c) $\frac{9}{2}\pi$
   (d) $12 + 4 + \frac{9}{2}\pi = 16 + \frac{9}{2}\pi$

1. A: $\int_0^5 f(t)\,dt$ greatest distance, B: $\int_0^5 g(t)\,dt$, C: $\int_0^5 h(t)\,dt$ least distance

4. (a) $L_4 = (0)^3 \frac{1}{2} + \left(\frac{1}{2}\right)^3 \frac{1}{2} + \left(\frac{2}{2}\right)^3 \frac{1}{2} + \left(\frac{3}{2}\right)^3 \frac{1}{2} = \frac{9}{4}$

$R_4 = \left(\frac{1}{2}\right)^3 \frac{1}{2} + \left(\frac{2}{2}\right)^3 \frac{1}{2} + \left(\frac{3}{2}\right)^3 \frac{1}{2} + \left(\frac{4}{2}\right)^3 \frac{1}{2} = \frac{25}{4}$

(b) $L_6 = \left(\frac{1}{\frac{3}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{4}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{5}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{6}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{7}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{8}{3}}\right)\frac{1}{3} = \frac{341}{280} \approx 1.218$

$R_6 = \left(\frac{1}{\frac{4}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{5}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{6}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{7}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{8}{3}}\right)\frac{1}{3} + \left(\frac{1}{\frac{9}{3}}\right)\frac{1}{3} = \frac{2509}{2520} \approx 0.9956$

5. Note: We assume that $v_g(t)$ is zero on [0, 1] and [7, 8].

(a) The area under $v_c(t)$ on [0, 1] which is $\int_0^1 v_c(t)dt$.

(b) The area under $v_c(t)$ minus area under $v_g(t)$ on [0, 1.5] is $\int_0^{1.5} v_c(t)dt - \int_0^{1.5} v_g(t)dt$.

(c) The absolute value of [the area under $v_c(t)$ minus area under $v_g(t)$] on [0, 3]

$= \left| \int_0^3 v_c(t)dt - \int_0^3 v_g(t)dt \right|$, (we don't know which quantity is largest).

(d) $t = 6.5$ $v_g(t)$ was $> v_c(t)$ so the goat kept getting further ahead. After $t = 6.5$ $v_g(t) < v_c(t)$.

(e) $t \approx 3$ Areas under velocity functions on [0, 3] are approximately equal.

(f) $t \approx 4.2$ The difference in velocities is the greatest at $t = 4.2$, $v_g'(4.2) = v_c'(4.2)$.

6. (a) $L_4 < L_{20} < L_{100} < \int_0^2 x^3 dx < R_{100} < R_{20} < R_4$.

(b) $|R_4 - L_4| = \left|2^3 \frac{2}{4} - 0^3 \frac{2}{4}\right| = 4 = \frac{2^4}{4}$.

(c) $|R_{100} - L_{100}| = \left|2^3 \frac{2}{100} - 0^3 \frac{2}{100}\right| = \frac{2^4}{100} = 0.16$

(d) $|R_n - L_n| = \left|2^3 \frac{2}{n} - 0^3 \frac{2}{n}\right| = \frac{2^4}{n} < 0.05 \Leftrightarrow \frac{2^4}{0.05} < n \Leftrightarrow 320 < n$

(e) $R_4 = \sum_{k=1}^{4} \left(\frac{k}{2}\right)^3 \frac{1}{2} = \left(\frac{1}{2}\right)^3 \frac{1}{2} + \left(\frac{2}{2}\right)^3 \frac{1}{2} + \left(\frac{3}{2}\right)^3 \frac{1}{2} + \left(\frac{4}{2}\right)^3 \frac{1}{2} = \frac{25}{4}$