Chapter 23

The Area Function and Its Characteristics

23.1 An Introduction to the Area Function $\int_a^x f(t)dt$

2. By looking at the area under the graph of $f(t) = 2t$ over the
    appropriate interval we get area of a triangle or trapezoid.

(a) $_0A_f(x) = \frac{1}{2}x(2x) = x^2$, $_0A_f(-3) = 9$ works.

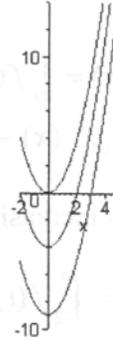   (Signed area under $f(t) = 2t$ is negative but the end points are
   interchanged to introduce another negative sign.)

(b) $_2A_f(x) = \frac{1}{2}(4+2x)(x-2) = x^2 - 4$, works for $x < 2$.

(c) $_3A_f(x) = \frac{1}{2}(6+2x)(x-3) = x^2 - 9$, works for $x < 3$.

(d) The graphs are shown to the right in order top to bottom.

(e) The derivative of all three functions is the same
    function $f(x) = 2x$.

4. (a) $_0A_f(x)$ is the <u>additional</u> number of gallons in the tank x minutes after noon.

   (b) $_0A_f(x)$ is increasing as long as $f(t) = 40 - 2t > 0$,   during interval $0 \le t \le 20$.

   (c) $_0A_f(40) = 0$ because at $t = 40$ the signed area under $f(t) = 40 - 2t$ is zero.
       (As much flowed out during [20, 40] as flowed in during [0, 20].)

23.2 Characteristics of the Area Function

1. (a) [0, 8] where $f(x) \ge 0$.

   (b) [8, 11] where $f(x) \le 0$.

   (c) $A_f(0) = 0$, there is zero area on [0, 0].

   (d) No. The signed area is always positive on [0, x] for x in [0, 11].

   (e) Absolute minimum at $x = 0$, global maximum at $x = 8$, local minimum at $x = 11$.

2. (a) B $_0A_f(0) = 0$. To the right of $x = 0$ the area increases more near to 0 than farther to the right.

   To the left $_0A_f(x) = \int_0^x f(t)dt = -\int_x^0 f(t)dt = -$ (negative signed area) = positive value.

   (b) D $_0A_f(x) = \int_0^x f(t)dt = \int_0^2 f(t)dt + \int_2^x f(t)dt = \int_0^2 f(t)dt + {_2}A_f(x) \Rightarrow$

   $_2A_f(x) = {_0}A_f(x) - (+$ number$)$ so $_2A_f(x)$ is $_0A_f(x)$ shifted down.