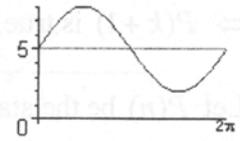
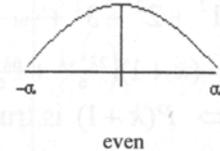
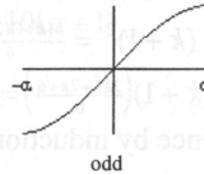


24.2 The Average Value of a Function: An Application of the Definite Integral

1. By the graph we can see that as much area above the line $y = 5$ is gained on $[0, \pi]$ as is lost on $[\pi, 2\pi]$. Hence $y = 5$ is an average value. Also $v_{ave} = \frac{1}{2\pi} \int_0^{2\pi} (3 \sin x + 5) dx = \frac{1}{2\pi} (-3 \cos x + 5x) \Big|_0^{2\pi}$
 $= \frac{1}{2\pi} [(-3(1) + 5(2\pi)) - (-3(1) + 5(0))] = 5$



2. (a) For the odd function we can see that the signed area on $[-a, a]$ will always be 0. Hence the average value will also be 0.



- (b) For the even function, by symmetry the average value on $[-a, a]$, $[-a, 0]$ and $[0, a]$ will all be the same.

5. (a) Velocity of bicyclist at 5 PM.
 (b) How many miles east from the starting point the bicyclist is at 5 PM.
 (c) Total distance in miles the bicyclist has traveled from noon to 5 PM.
 (d) Speed of bicyclist at 5 PM.
 (e) Acceleration of bicyclist at 5 PM.
 (f) Average velocity of bicyclist from noon to 5 PM.

9. distance = (average velocity)(time)

(a) $D_{0 \rightarrow 2} = (14)(2) = 28$ miles, $D_{2 \rightarrow 6} = (18)(4) = 72$, so $D_{0 \rightarrow 6} = 100$

Hence $v_{ave} = \frac{100}{6} = \frac{50}{3}$ miles per hour.

(b) $D_{0 \rightarrow 2} = (14)(2) = 28$ miles, $D_{2 \rightarrow 6} = (18)(4) = 72$, so $D_{0 \rightarrow 6} = 100$

13.(a) $\int_0^{36} B(t) dt$

(b) $\frac{1}{10} \int_{20}^{30} D(t) dt$

(c) $4500 + \int_0^{36} B(t) dt - \int_0^{36} D(t) dt$

(d) 1972 ($t = 12$). Birth rate has been greater than death rate but switches at $t = 12$.

(e) Less, as had more deaths than births. $\int_0^{36} B(t) dt - \int_0^{36} D(t) dt < 0$