2 9. Rewrite and then use L'Hopital's rule: $\lim\limits_{x\to\infty} 2x\cdot e^{-x} = \lim\limits_{x\to\infty} \frac{2x}{e^x} = \lim\limits_{x\to\infty} \frac{2}{e^x} = 0$.

2 19. $\lim\limits_{x\to 0^+} e^x \ln x = -\infty$ since it is of type $1\cdot(-\infty)$.

2 20. Rewrite $\left(1+\frac{5}{x}\right)^{3x} = \left(\left(1+\frac{5}{x}\right)^{\frac{x}{5}}\right)^{15}$ and substitute $k = \frac{x}{5}$,

then $\lim\limits_{x\to\infty}\left(1+\frac{5}{x}\right)^{3x} = \lim\limits_{x\to\infty}\left(\left(1+\frac{5}{x}\right)^{\frac{x}{5}}\right)^{15} = \lim\limits_{k\to\infty}\left(\left(1+\frac{1}{k}\right)^{k}\right)^{15} = e^{15}$.

2 21. Rewrite and then use L'Hopital's rule: $\lim\limits_{x\to\infty} e^{-x}\ln x = \lim\limits_{x\to\infty} \frac{\ln x}{e^x} = \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{e^x} = \lim\limits_{x\to\infty} \frac{1}{x\cdot e^x} = 0$

2 22. $f(x) = x^2 \ln x \Rightarrow f'(x) = 2x\ln x + x^2\frac{1}{x} = x(2\ln x + 1)$. Critical points occur when $x = 0$ or $2\ln x + 1 = 0 \Rightarrow x = 0$ or $x = e^{-1/2} \approx 0.60653$. A local minimum occurs when $x = e^{-1/2}$. $x = 0$ is not in the domain of f.
Using L'Hopital's rule $\lim\limits_{x\to 0^+} x^2\ln x = \lim\limits_{x\to 0^+} \frac{\ln x}{x^{-2}} = \lim\limits_{x\to 0^+} \frac{1/x}{-2x^{-3}} = \lim\limits_{x\to 0^+} \frac{x^2}{-2} = 0$. The graph shows the height approaching zero as x approaches zero from the right.