Math Xb Spring 2004

Lab 2: Related Rates

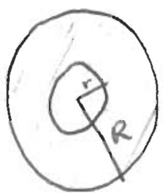
February 19, 2004

Name SOLUTIONS

Section Instructor

Collaborators

1. Two concentric circles are expanding, the outer radius at a rate of 2 feet per second and the inner one at a radius of 5 feet per second. At a certain instant, the outer radius is 10 feet, and the inner radius is 3 feet. At this instant, is the area of the ring between the two circles increasing or decreasing? Justify your answer.



r = radius of inner circle

R = radius of outer circle

A = area between the circles

$\frac{dR}{dt} = 2$ ft/s $\quad \frac{dr}{dt} = 5$ ft/s

$\frac{dA}{dt} = ?$ when $R = 10$ ft, $r = 3$ ft

$A = \pi R^2 - \pi r^2$

$\frac{dA}{dt} = 2\pi R \frac{dR}{dt} - 2\pi r \frac{dr}{dt}$

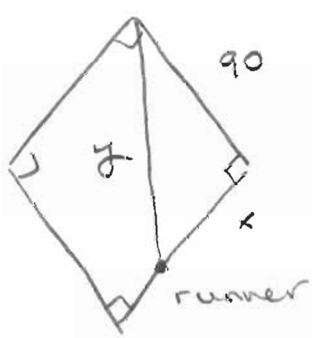
$= 2\pi(10)(2) - 2\pi(3)(5)$

$= 40\pi - 30\pi$

$= 10\pi \approx \boxed{31.42 \text{ ft}^2/\text{s}}$

2. A baseball diamond is a square with side 90 feet. The corners of the square are labeled home plate, first base, second base, and third base as one moves around the square counter-clockwise. A batter hits the ball and runs from home plate towards first base with a speed of 24 feet per second.

   (a) At what rate is his distance from second base changing when he is halfway to first base?



x = distance between runner and 1st base

y = distance between runner and 2nd base

$\frac{dx}{dt} = -24$ ft/s (since x is decreasing)

$\frac{dy}{dt} = ?$ when $x = 45$ ft

$x^2 + 90^2 = y^2$

$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{x \frac{dx}{dt}}{y}$

Need to find y:

$45^2 + 90^2 = y^2$

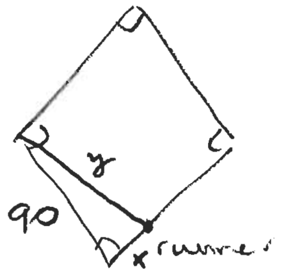
$y = \sqrt{10125}$

≈ 100.6

$\Rightarrow \frac{dy}{dt} \approx \frac{45(-24)}{100.6}$

$\approx \boxed{-10.73 \text{ ft/s}}$

(b) At what rate is his distance from third base changing at the same moment?



x = distance between runner and home plate

y = distance between runner and 3^{rd} base

$\frac{dx}{dt} = 24$ ft/s $\qquad \frac{dy}{dt} = ?$ when $x = 45$ ft

$$x^2 + 90^2 = y^2$$

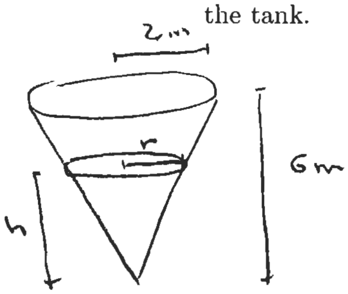
$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{x\frac{dx}{dt}}{y}$$

$y \approx 100.6$ again

$$\Rightarrow \frac{dy}{dt} = \frac{45(24)}{100.6} \approx \boxed{10.73 \text{ ft/s}}$$

3. Water is leaking out of an inverted conical tank at a rate of 10,000 cubic centimeters per minute at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 meters and the diameter at the top is 4 meters. If the water level is rising at a rate of 20 centimeters per minute when the height of the water is 2 meters, find the rate at which water is being pumped into the tank.



r = radius of cone of water

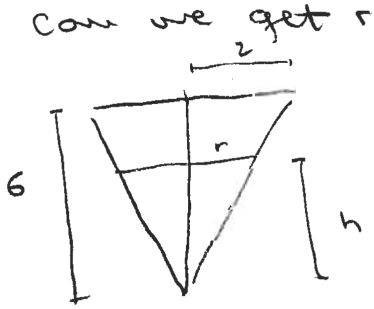
h = height of cone of water

V = volume of cone of water

$\frac{dV}{dt} = ?$ when $\frac{dh}{dt} = 20 \frac{\text{cm}}{\text{min}}$ and $h = 200$ m

$$V = \frac{1}{3}\pi r^2 h$$

Can we get rid of r?



$$\Rightarrow \frac{r}{h} = \frac{2}{6}$$

$$\Rightarrow r = \frac{1}{3}h$$

$$\Rightarrow V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$$

$$V = \frac{1}{27}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$$

$$= \frac{1}{9}\pi(200)^2(20) \approx 279{,}000$$

$$\frac{dV}{dt} = \text{in rate} - \text{out rate}$$

$$279{,}000 = \text{in rate} - 10{,}000$$

$$\Rightarrow \text{in rate} = \boxed{289{,}000 \frac{\text{cm}^3}{\text{min}}}$$