Math Xb Spring 2004 Name solutions

Lab 5: Modeling with Trig Functions Section Instructor

March 9, 2004 Collaborators

1. In Boston, the number of hours of daylight $D(t)$ at a particular time of the year may be approximated by
$$D(t) = 3\sin\left(\frac{2\pi}{365}(t-79)\right) + 12,$$
with t in days and $t = 0$ corresponding to January 1.

(a) What is the maximum number of hours of daylight Boston receives during the year?

max of sin func = 1 (when $t \approx 170$). So max $D(t) = 3+12 = 15$

(b) What is the minimum number of hours of daylight Boston receives during the year?

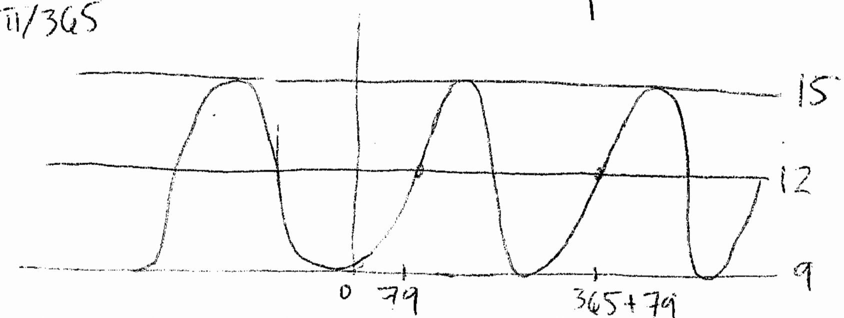
min of sin func = −1 (when $t \approx 352$). So min $D(t) = -3+12 = 9$

(c) What is the average number of hours of daylight Boston receives during the year?

average = balance value of D = 12

(d) Sketch the graph of $D(t)$. You may use your calculator to assist you, but the graph you sketch here should make clear the amplitude, balance value, and period of $D(t)$.

period $= \frac{2\pi}{B} = \frac{2\pi}{2\pi/365} = 365$ amplitude = 3



(e) How many days of the year have more than 10.5 hours of daylight?

$D(t) = 3\sin\left(\frac{2\pi}{365}(t-79)\right) + 12 > 10.5 \Rightarrow$

$\sin\left(\frac{2\pi}{365}(t-79)\right) > \frac{-1.5}{3} = -\frac{1}{2}$

So $\frac{2\pi}{365}(t-79) = \frac{-\pi}{6} \Rightarrow t = 49$

$\frac{2\pi}{365}(t-79) = \frac{7\pi}{6} \Rightarrow t = 291$

So $291 - 49 = 242$ days have > 10.5 hrs of daylight.

2. When an earthquake occurs along a fault line, *slip* is the relative displacement of formerly adjacent points on opposite sides of a fault, measured along the fault line. After the 1906 San Francisco earthquake, the slip $S(d)$ in meters of a point d kilometers from the earthquake's fault line could be approximated by

$$S(d) = 2\left(1 - \frac{2}{\pi}\tan^{-1}\left(\frac{d}{3.5}\right)\right).$$

(a) Estimate the slip of a point 1 kilometer from the earthquake's fault line.

$S(1) \approx 1.65$ meters

(b) According to this model, how far did a point on the earthquake's fault line slip?

$\tan^{-1}(0) = 0$, so $S(0) = 2$ Meters

(c) If it is determined that a point slipped approximately 0.92 meters during the earthquake, approximately how far from the earthquake's fault line was the point?

$$.92 = 2 - \frac{4}{\pi}\tan^{-1}\left(\frac{d}{3.5}\right) \Rightarrow \frac{.92-2}{-\frac{4}{\pi}} = \tan^{-1}\left(\frac{d}{3.5}\right)$$

$$\tan\left((2-.92)\cdot\frac{\pi}{4}\right) = \frac{d}{3.5}$$

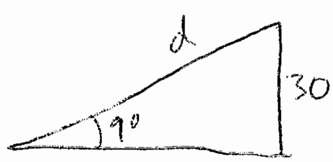
$d \approx 4$ KM

(d) Find $\lim_{d\to\infty} S(d)$ and interpret your answer in words.

$\lim_{t\to\infty} \tan^{-1}(t) = \pi/2$, so $\lim_{d\to\infty} S(d) = 0$

A point infinitely far from the fault line does not slip at all

3. Stonehenge in Salisbury Plains, England, was constructed using solid stone blocks weighing over 99,000 pounds each. Lifting a single stone required 550 people, who pulled the stone up a ramp inclined at an angle of 9°. Approximate the distance that a stone was moved in order to raise it to a height of 30 feet.



$\sin(9°) = \frac{30}{d}$

$\boxed{192 \text{ Feet}}$