

1. Find each of the following limits.

$$(a) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{10}\right)^x = \lim_{x \rightarrow \infty} (1.1)^x = \infty \quad \text{if } B > 1, \lim_{x \rightarrow \infty} (B)^x = \infty$$

$$(b) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{100}\right)^x = \lim_{x \rightarrow \infty} (1.01)^x = \infty$$

$$(c) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{1000}\right)^x = \lim_{x \rightarrow \infty} (1.001)^x = \infty$$

$$(d) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{10000}\right)^x = \lim_{x \rightarrow \infty} (1.0001)^x = \infty$$

2. Based on your answers to Question 1, what does $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$ appear to be?

3. Find each of the following limits.

$$(a) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{10} = 1$$

$$(b) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{100} = 1$$

$$(c) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{1000} = 1$$

$$(d) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{10000} = 1$$

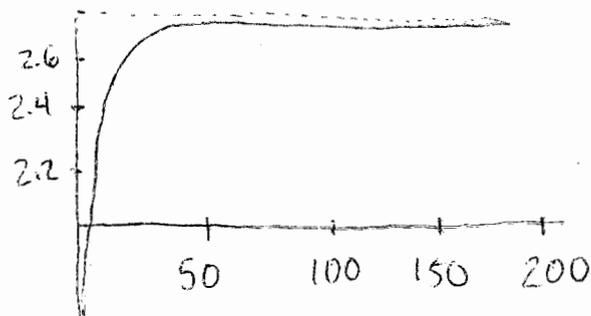
4. Based on your answers to Question 3, what does $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$ appear to be? 1

5. The expression $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$ is said to be of indeterminate form 1^∞ . Given your answers to the previous questions, explain in words why something of the form 1^∞ is indeterminate.

base getting smaller } so we cannot tell whether
exponent getting larger } the limit is 1 or ∞ .

6. Graph the function $f(x) = \left(1 + \frac{1}{x}\right)^x$ on your graphing calculator on the domain $[0, 100]$.

(a) Sketch the graph in the space below. Make clear any important features of the graph.



(b) Does $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$ appear to exist? **YES**

7. Use your calculator to complete the following table of values for $f(x) = \left(1 + \frac{1}{x}\right)^x$. Give your answers in decimal form, accurate to eight decimal places.

x	$f(x)$
10	2.59374246
100	2.70481382
1,000	2.71692393
10,000	2.71814592
100,000	2.71826823

8. Given your answers to the previous two questions, $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$ is probably equal to what popular mathematical constant? **e**

9. Does the series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$ converge or diverge? Explain the reasoning behind your answer. (Note that the series is *not* geometric, but we can still determine if it converges or diverges.)

diverges: terms approach e, so this is eventually like adding e to itself an ∞ number of times.