**Math Xb Spring 2004** Name SOLUTIONS

**Lab 10: Differential Equations** Section Instructor ____

**May 4, 2004** Collaborators ____

In a laboratory, a colony of fruit flies are under scrutiny. Let $P = P(t)$ be the number of fruit flies in the colony at time $t$.

Given ample food, the population would grow at a rate proportional to itself according to $\frac{dP}{dt} = kP$. But flies are continually being siphoned off to another lab at a constant rate of $C$ flies per day.

The rate of change of the fly population is the rate at which it is increasing due to reproduction minus the rate at which flies are being siphoned off.

We model the situation as follows.

$$\text{(Rate of change)} = \text{(Rate of increase)} - \text{(Rate of decrease)}$$

$$\frac{dP}{dt} = kP - C$$

We want to solve this differential equation for $P(t)$.

1. Suppose $k = 0.02$ and $C = 2$. Then we want to solve the differential equation

$$\frac{dP}{dt} = 0.02P - 2.$$

First, rewrite it as

$$\frac{dP}{dt} = 0.02(P - 100).$$

We can convert this differential equation to a form with which we are familiar by making the substitution $y = P - 100$.

(a) Express the differential equation $\frac{dP}{dt} = 0.02(P - 100)$ in terms of $y$.

$$\frac{dy}{dt} = \frac{dP}{dt}$$

$$\Rightarrow \frac{dy}{dt} = 0.02y$$

(b) Solve the differential equation you found in part (a).

$$y(t) = y_0 e^{0.02t}$$

(c) Knowing that $y = P - 100$, find $P(t)$. There will be an arbitrary constant in your answer because you will have found the general solution to the differential equation.

$$y_0 e^{0.02t} = y(t) = P(t) - 100$$

$$\Rightarrow P(t) = y_0 e^{0.02t} + 100$$

(d) Suppose that $P(0) = 3000$. Find the particular solution corresponding to this initial condition.

$$3000 = P(0) = y_0 e^{0.02(0)} + 100$$

$$y_0 = 2900$$

$$\Rightarrow \boxed{P(t) = 2900e^{0.02t} + 100}$$

2. Use similar substitution techniques to solve the following differential equations.

(a) $\frac{dQ}{dt} = 2Q - 6$ $= 2(Q-3)$

Let $y = Q - 3$. Then $\frac{dy}{dt} = \frac{dQ}{dt} = 2(Q-3) = 2y$

$\Rightarrow y(t) = y_0 e^{2t}$

$\Rightarrow$ $\boxed{Q(t) = y_0 e^{2t} + 3}$

(b) $\frac{dM}{dt} = 0.1M - 200$ $= 0.1(M - 2000)$

Let $y = M - 2000$.

Then $\frac{dy}{dt} = \frac{dM}{dt} = 0.1(M-2000) = 0.1y$

$\Rightarrow y(t) = Ce^{0.1t}$

$\Rightarrow$ $\boxed{M(t) = Ce^{0.1t} + 2000}$