Math Xb Spring 2004
Worksheet: L'Hôpital's Rule Day Two
March 24, 2004

1. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, then the value of the investment after t years is

$$A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}.$$

   (a) Suppose \$1000 is invested in an account bearing 5% interest compounded annually. How much will the account be worth after 1 year?

$$A(1) = \$1000\left(1 + \frac{.05}{1}\right)^1 = \$1050$$

   (b) Suppose the interest is compounded monthly instead. How much will the account be worth after 1 year?

$$A(1) = \$1000\left(1 + \frac{.05}{12}\right)^{12} = \$1051.16$$

   (c) Suppose the interest is compounded daily instead. How much will the account be worth after 1 year?

$$A(1) = \$1000\left(1 + \frac{.05}{365}\right)^{365} = \$1051.27$$

d) let $L = \lim_{n\to\infty} \left(1 + \frac{.05}{n}\right)^n$

$$\ln(L) = \lim_{n\to\infty} n \ln\left(1+\frac{.05}{n}\right) = \lim_{n\to\infty} \frac{\ln\left(1+\frac{.05}{n}\right)}{1/n}$$, use L-Hôpital:

$$\ln(L) = \lim_{n\to\infty} \frac{\left(\frac{1}{1+\frac{.05}{n}}\right)\cdot\left(\frac{-.05}{n^2}\right)}{-1/n^2} = \lim_{n\to\infty}\left(\frac{-.05}{n^2+.05n}\right)\cdot -n^2 =$$

$$\lim_{n\to\infty} \frac{.05n^2}{n^2+.05n} = \lim_{n\to\infty} \frac{.05n}{n+.05} = .05$$

So $L = e^{.05} = 1.05127$

$1000L = \boxed{\$1051.27} =$ value of account

e) let $L = \lim_{n\to\infty}\left(1+\frac{r}{n}\right)^{n\cdot t}$. $\ln L = \lim_{n\to\infty} nt \ln\left(1+\frac{r}{n}\right) =$

$$\lim_{n\to\infty} \frac{\ln\left(1+\frac{r}{n}\right)}{1/nt}$$, use l'Hôpital: $= \lim_{n\to\infty} \frac{\frac{1}{\left(1+\frac{r}{n}\right)}\cdot\left(\frac{-r}{n^2}\right)}{-\frac{1}{n^2 t}} =$

$$\lim_{n\to\infty} \frac{-r}{\left(1+\frac{r}{n}\right)}\cdot\frac{1}{n^2} \div -n^2 t = \lim_{n\to\infty} \frac{t\cdot r}{\left(1+\frac{r}{n}\right)} = r.t.$$

So $L = e^{rt}$. Then the value of the account, $A(t) = A_0 \cdot e^{rt}$.

2. Given that

$$\lim_{x \to a} f(x) = 0 \qquad \lim_{x \to a} g(x) = 0 \qquad \lim_{x \to a} h(x) = 1$$

$$\lim_{x \to a} p(x) = \infty \qquad \lim_{x \to a} q(x) = \infty$$

which of the following are indeterminate forms? For those that are not an indeterminate form, evaluate the limit where possible.

(a) $\lim_{x \to a} f(x)p(x)$ $= 0 \cdot \infty =$ indeterminate

(b) $\lim_{x \to a} h(x)p(x)$ $= 1 \cdot \infty = \infty$

(c) $\lim_{x \to a} p(x)q(x)$ $= \infty^2 = \infty$

(d) $\lim_{x \to a} f(x)^{g(x)}$ $= 0^0 =$ indeterminate

(e) $\lim_{x \to a} f(x)^{p(x)}$ $= 0^\infty = 0$

(f) $\lim_{x \to a} h(x)^{p(x)}$ $= 1^\infty = 1$

(g) $\lim_{x \to a} p(x)^{f(x)}$ $= \infty^0 =$ indeterminate

(h) $\lim_{x \to a} p(x)^{q(x)}$ $= \infty^\infty = \infty$