Problem 4.

(a) $y(t) = Ce^{-2t}$. $\sqrt{2} = Ce^{-2(0)} \Rightarrow C = \sqrt{2} \Rightarrow y(t) = \sqrt{2}e^{-2t}$.

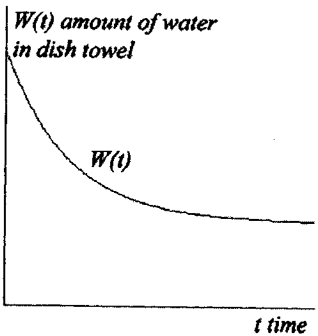
(b) $y(t) = -t^2 + C$. $\sqrt{2} = -(0)^2 + C \Rightarrow C = \sqrt{2} \Rightarrow y(t) = -t^2 + \sqrt{2}$.

(c) $y(t) = -2t + C$. $\sqrt{2} = -2(0) + C \Rightarrow C = \sqrt{2} \Rightarrow y(t) = -2t + \sqrt{2}$.

Problem 8.

(a) $\frac{dW}{dt} = k(W - M)$, for some constant $k < 0$.

(b) Let $D(t)$ be the difference between the moisture in the rag and in the air. Now $D(t) = W(t) - M$ and $\frac{dD}{dt} = \frac{dW}{dt}$. In terms of D, the differential equation becomes $\frac{dD}{dt} = kD$. Hence $D(t) = Ce^{kt}$ for an arbitrary constant C. Substituting $D(t) = W(t) - M$, we have $W(t) = Ce^{kt} + M$.



Problem 9.

(a) We have $\frac{dM}{dt} = kM$ and $250 = k(5000)$. Hence $k = 0.05$, and $\frac{dM}{dt} = 0.05M$.

(b) We have $\frac{dB}{dt} = kB$. The general solution is $B(t) = Ce^{kt}$. We know that $600 = Ce^{k(0)}$, and hence $C = 600$. We also know that $800 = 600e^{k(10)}$, and hence $k = \frac{\ln(\frac{4}{3})}{10} \approx 0.0288$. Therefore $\frac{dB}{dt} = 600e^{(\ln(4/3))/10} = 600\left(\frac{4}{3}\right)^{1/10}$.

Problem 12.

$\frac{dM}{dt} = 0.05M - 2000$ and $M(0) = 30,000$.