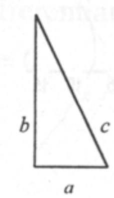
17.4 Implicit Differentiation in Context: Related Rates of Change

1. $A=\pi r^2 \Rightarrow \frac{dA}{dt}=\pi 2r\frac{dr}{dt}$. At $d=4,\ r=2$ so $\frac{dA}{dt}=\pi 2(2)(3)=12\pi\ {}^{ft^2}/_{sec}$

6. $\frac{db}{dt}=-40\frac{m}{hr},\ \frac{da}{dt}=60\frac{m}{hr},\ a^2+b^2=c^2$, when $t=\frac{1}{2}$,

$b=100-40(\frac{1}{2})=80,\ a=60(\frac{1}{2})=30,\ c^2=80^2+30^2 \Rightarrow c=10\sqrt{73}$

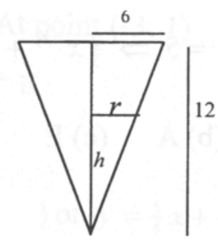
$2c\frac{dc}{dt}=2a\frac{da}{dt}+2b\frac{db}{dt} \Rightarrow \frac{dc}{dt}=\frac{a\frac{da}{dt}+b\frac{db}{dt}}{c}=\frac{30(60)+80(-40)}{10\sqrt{73}}=\frac{-140}{\sqrt{73}}\frac{m}{hr}\approx -16.4\frac{m}{hr}$



8. $V=\frac{1}{3}\pi r^2 h$. By similar triangles $\frac{h}{r}=\frac{12}{6} \Rightarrow h=2r$ so $r=\frac{1}{2}h$

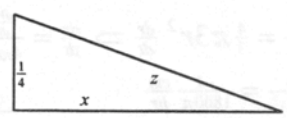
Substituting gives $V=\frac{1}{3}\pi(\frac{1}{2}h)^2 h=\frac{1}{12}\pi h^3$

$\Rightarrow \frac{dV}{dt}=\frac{1}{12}\pi 3h^2\frac{dh}{dt} \Rightarrow -2=\frac{1}{4}\pi(10)^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt}=\frac{-2}{25\pi}\frac{cm}{sec}$



10. $z^2=\left(\frac{1}{4}\right)^2+x^2$, in $\frac{1}{3}$ hr $x=5 \Rightarrow z^2=\left(\frac{1}{4}\right)^2+5^2=\frac{401}{16}$

$2z\frac{dz}{dt}=2x\frac{dx}{dt} \Rightarrow \frac{dz}{dt}=\frac{x}{z}\frac{dx}{dt} \Rightarrow \left.\frac{dz}{dt}\right|_{x=5}=5\frac{4}{\sqrt{401}}15\approx 14.98mph$



13. $h=\frac{1}{2}r \Rightarrow r=2h$. Substituting gives $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi(2h)^2 h=\frac{4}{3}\pi h^3$

(a) $\frac{dV}{dt}=4\pi h^2\frac{dh}{dt} \Rightarrow 5=4\pi(9)^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt}=\frac{5}{324\pi}\approx 0.0049\frac{m}{min}$

(b) $A=\pi r^2=\pi(2h)^2=4\pi h^2 \Rightarrow \frac{dA}{dt}=8\pi h\frac{dh}{dt}=8\pi(9)(\frac{5}{324\pi})\approx 1.1111\frac{m^2}{min}$

(c) $C=2\pi r=4\pi h \Rightarrow \frac{dC}{dt}=4\pi\frac{dh}{dt}=4\pi(\frac{5}{324\pi})=\frac{5}{81}\approx 0.0617\frac{m}{min}$

(d) For the first equation in part (a) if $\frac{dV}{dt}$ is constant then the right side must be constant. As time goes on h will be getting larger so $\frac{dh}{dt}$ will need to be getting smaller. Hence the height will be increasing more slowly as time goes on.