

1. geometric $a = 1$ $r = -10$ 18. $a = 1$, $r = -10$, $S - (-10)S = 1 - (-10)^{11} \Rightarrow S = 9090909091$
4. geometric $a = 2/3$ $r = 1/3$ 24. $a = 1/e$, $r = 2/e$, $S - \frac{2}{e}S = \frac{1}{e} - \frac{2^{n+1}}{e^{n+2}} \Rightarrow S = \frac{1 - (\frac{2}{e})^{n+1}}{e - 2}$
1. geometric $a = -a$ $r = -a$ 27. $a = 1$, $r = 9/10$, $S - (\frac{9}{10})S = 1 - (\frac{9}{10})^{n+1} \Rightarrow S = 10(1 - (\frac{9}{10})^{n+1})$
12. geometric $a = 1/e$ $r = 2/e$ 32. $a = 1/x$, $r = -1/x$, $S - (-\frac{1}{x})S = \frac{1}{x} - \frac{1}{x^{11}} \Rightarrow S = \frac{\frac{1}{x} - \frac{1}{x^{11}}}{1 - (-\frac{1}{x})} = \frac{x^{10} - 1}{x^{10}(x+1)}$

18.2. Infinite Geometric Series

3. $r = \frac{1}{3}$, $a = \frac{2}{3}$, \Rightarrow converges to $\frac{a}{1-r} = \frac{\frac{2}{3}}{1 - (\frac{1}{3})} = 1$
4. $r = 3 \Rightarrow |r| > 1 \Rightarrow$ diverges
11. $r = \frac{1}{2e}$, $a = \frac{1}{(2e)^2}$, \Rightarrow converges to $\frac{\frac{1}{(2e)^2}}{1 - \frac{1}{2e}} = \frac{1}{4e^2 - 2e}$
6. $r = -\frac{1}{2}$, $a = \frac{1}{4}$, \Rightarrow converges to $\frac{\frac{1}{4}}{1 - (-\frac{1}{2})} = \frac{1}{6}$
8. $r = -\frac{1}{2}$, $a = \frac{3}{2}$, \Rightarrow converges to $\frac{\frac{3}{2}}{1 - (-\frac{1}{2})} = 1$

18.4 Summation Notation

7. (a) $\sum_{k=1}^{\infty} \frac{2}{3^k}$ (b) $\sum_{k=0}^{\infty} \frac{2}{3} 3^k$ 11. (a) $-\frac{1}{3} + \frac{1}{9}$, geometric with $r = -\frac{1}{3}$ hence converges to $\frac{-\frac{1}{3}}{1 - (-\frac{1}{3})} = -\frac{1}{4}$
8. (a) $\sum_{k=0}^{\infty} (-\frac{1}{2})^k$ (b) $\sum_{k=2}^{\infty} (-\frac{1}{2})^k$ (b) $\frac{1}{9} + (-\frac{1}{27})$, geometric with $r = -\frac{1}{3}$ hence converges to $\frac{\frac{1}{9}}{1 - (-\frac{1}{3})} = \frac{1}{12}$
20. (a) $\sum_{k=1}^{20} 500(e^{0.1})^k = \frac{500 - 500e^{2.1}}{1 - e^{0.1}}$ (b) $\sum_{k=0}^{\infty} \frac{5}{3} (-\frac{1}{2})^k = \frac{\frac{5}{3}}{1 - (-\frac{1}{2})} = \frac{10}{9}$

18.5 Applications of Geometric Sums and Series

6. $10000 = \frac{P}{(1.06)} + \frac{P}{(1.06)^2} + \dots + \frac{P}{(1.06)^8} = \frac{P(\frac{1}{1.06} - (\frac{1}{1.06})^9)}{1 - (\frac{1}{1.06})} = P(6.20979) \Rightarrow P \approx \1610.36

8. $PV = \frac{1000}{(1.06)} + \frac{1000}{(1.06)^2} + \dots + \frac{1000}{(1.06)^{10}} = \frac{1000(\frac{1}{1.06} - (\frac{1}{1.06})^{11})}{1 - (\frac{1}{1.06})} = \7360.09

12. $FV = 200(1 + \frac{0.045}{12})^{35} + 200(1.00375)^{34} + \dots + 200 = \frac{200(1 - (1.00375)^{36})}{1 - (1.00375)} = \7693.22

13. (a) $\frac{1}{2}(3) = 3e^{k(7)} \Rightarrow \ln(\frac{1}{2}) = k(7) \Rightarrow k = \frac{\ln(0.5)}{7} \approx -0.099021 \Rightarrow T \approx 3e^{(-0.099)t}$

(b) $A = 3e^{(-0.099)4} + 3e^{(-0.099)3} + \dots + 3 = \frac{3(1 - e^{(-0.099)5})}{1 - e^{(-0.099)1}} \approx 12.426$

(c) *Before* $= 3e^{(-0.099)30} + 3e^{(-0.099)29} + \dots + 3e^{(-0.099)1} = \frac{3(e^{(-0.099)1} - e^{(-0.099)31})}{1 - e^{(-0.099)1}} \approx 27.344$
After $= \text{Before} + 3 \approx 30.344$

(d) *Long Term* $= 3 + 3e^{(-0.099)1} + 3e^{(-0.099)2} + \dots = \frac{3}{1 - e^{(-0.099)}} \approx 31.821$