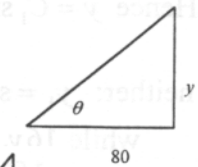
21.3 Applications

13. $y = 80\tan\theta \Rightarrow \frac{dy}{dt} = 80\sec^2\theta\,\frac{d\theta}{dt}$. At the instant the runner is 100ft away

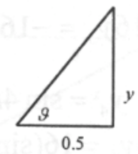
$\cos\theta = \frac{80}{100} = \frac{4}{5}$. Hence $9 = 80\frac{25}{16}\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = 0.072\,\frac{rad}{sec} \cong 0.01146\,\frac{rev}{sec}$



15. $\frac{y}{0.5} = \tan\theta \Rightarrow y = 0.5\tan\theta \Rightarrow \frac{dy}{dt} = 0.5\sec^2\theta\,\frac{d\theta}{dt}$

At the instant the window is one kilometer from the light

$\cos\theta = \frac{1}{2} \Rightarrow \sec^2\theta = 4 \Rightarrow \frac{dy}{dt} = 0.5(4)[6(2\pi)] = 24\pi \cong 75.4\,\frac{km}{min}$



17.(a) $D(x) = \sin x - (-\cos x) \Rightarrow D'(x) = \cos x - \sin x = 0 \Leftrightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

Hence the maximum distance is $D(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

(b) Tangent lines to these two curves are parallel at $\frac{\pi}{4}$. This is not surprising as if they were not parallel, then movement in one of the directions would make the curves move further apart.

21.4

6. $y = \frac{1}{2}\arctan(e^x) \Rightarrow y' = \frac{1}{2}\frac{1}{1+(e^x)^2}e^x = \frac{e^{(x-1)}}{1+e^{2x}}$

8. $f'(x) = 3\left(-\sin\left(\frac{1}{1+x^2}\right)(-1(1+x^2)^{-2}(2x)) + (1)\arctan\left(\frac{1}{x}\right) + x\frac{1}{1+(\frac{1}{x})^2}(-1x^{-2}\right)$

$= \frac{6x}{(1+x^2)^2}\sin\left(\frac{1}{1+x^2}\right) + \arctan\left(\frac{1}{x}\right) - \frac{x}{x^2+1}$

9. $\frac{d}{dx}\left(\frac{\sin^{-1}x}{\cos^{-1}x}\right) = \frac{\frac{1}{\sqrt{1-x^2}}\cos^{-1}x - \sin^{-1}x\frac{-1}{\sqrt{1-x^2}}}{(\cos^{-1}x)^2}$ which is not equal to $\frac{1}{1+x^2}$