f The area under the curve is close to a triangle of base 9 and height 2

$c < d < a < b < e = f$

(a) $x^2 + y^2 = 2^2$ or $x^2 + y^2 = 4$

(b) $y = \sqrt{4 - x^2} = f(x)$ on $[-2,2]$

(c) $\int_{-2}^{2} f(x)dx = \int_{-2}^{0} f(x)dx + \int_{0}^{2} f(x)dx = \int_{-2}^{0}\sqrt{4-x^2}\,dx + \int_{0}^{2} 2x dx$ = area of a quarter circle of radius 2 plus the area of a triangle with base 2 and height 4 $= \frac{1}{4}\pi\ 2^2 + \frac{1}{2}(2)(4) = \pi + 4$

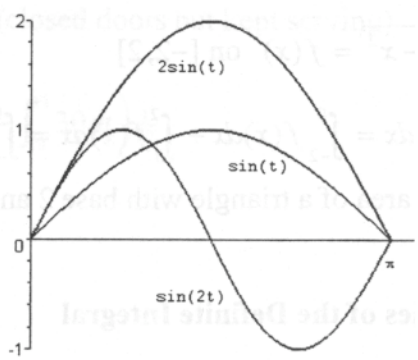
4

a) $\int_{0.5}^{3} \frac{1-\ln x}{x^2+1}\,dx > \int_{0.5}^{3} \frac{1-\ln x}{x^2+1}\,dx + \int_{3}^{4} \frac{1-\ln x}{x^2+1}\,dx = \int_{0.5}^{4} \frac{1-\ln x}{x^2+1}\,dx$, because $\int_{3}^{4} \frac{1-\ln x}{x^2+1}\,dx$ is negative on [3,4]. $\ln x > 1$ on that interval so $\frac{1-\ln x}{x^2+1}$ is negative on that interval.

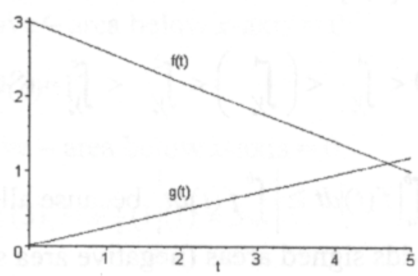
b) $0 < \int_{1/e}^{1} < \left(\int_{1/e}^{4}\right) < \int_{1/e}^{2} < \int_{1/e}^{e}$ (Students shouldn't know, at this stage, how small $\int_{1/e}^{4}$ is.)

$\int_{\pi}^{0} \sin t\,dt = -\int_{0}^{\pi} \sin t\,dt$.

$\int_{\pi}^{0} \sin t\,dt < \int_{0}^{\pi} \sin(2t)\,dt < \int_{0}^{\pi} \sin t\,dt < \int_{0}^{\pi} 2\sin t\,dt$



9. (a) No! Use any example with proper areas and g having greater output than f somewhere in the domain. For instance let $f(t) = 3 - \frac{2}{5}t$ and $g(t) = \frac{6}{25}t$, then $\int_0^5 f(t)dt = 10$ *and* $\int_0^5 g(t)dt = 3$, but $f(5) = 1$, while $g(5) = \frac{6}{5}$



(b) Yes! If not, the area under $f(t)$ couldn't possibly be larger than the area under $g(t)$.