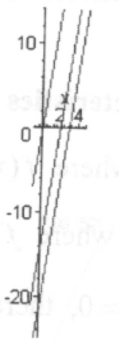
23.1 An Introduction to the Area Function

1. (a) ${}_0A_f(x) = \int_0^x f(t)dt = \int_0^x 7dt = 7(x-0) = 7x,$
   ${}_0A_f(-3) = 7(-3) = -21$ works.

   (b) ${}_2A_f(x) = \int_2^x f(t)dt = \int_2^x 7dt = 7(x-2) = 7x-14,$
   which works for $x < 2$ also.

   (c) ${}_3A_f(x) = \int_3^x f(t)dt = \int_3^x 7dt = 7(x-3) = 7x-21,$
   which works for $x < 3$ also.

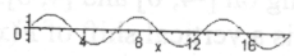
   (d) The graphs are shown to the right in order left to right.



   (e) The derivative of all three functions is the same constant function $f(x) = 7$.

3. $A_f(x)$ is the signed area on interval $[0, x]$.

   (a) $A_f(0) = A_f(2\pi) = 0 < A_f(\frac{\pi}{2}) = A_f(\frac{3\pi}{2})$



   (b) $A_f(x) = 0$ when signed area = 0. $x = k2\pi,\ k\varepsilon Z.$

   (c) $A_f(x)$ is never negative.

   (d) $A_f(x)$ is max. when $x = \pi + k2\pi = (2k+1)\pi,\ k\varepsilon Z.$

23.2

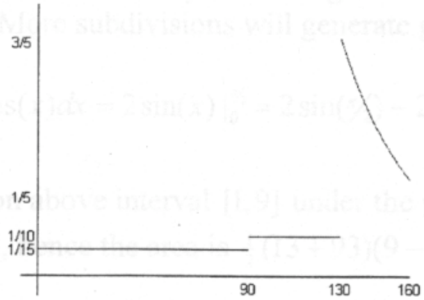
3. (a) C $F(x)$ is increasing on [-1, 2], and [5, 6]; decreasing on [-2, -1] and [2, 5].

   (b) C $G(x) = \int_{-2}^{0} f(t)dt + F(x)$ so is a vertical shift of $F(x)$ with $G(-2) = 0$.

23.3

4. Approximate graphs are given below with time units in minutes.

   (a)



   (b)