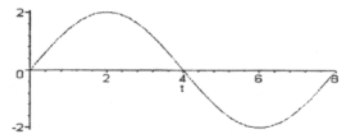
24.1 Definite Integrals and the Fundamental Theorem

7. $\int_0^9 (1.5t+\sqrt{t})dt = \int_0^9 (\frac{3}{2}t + t^{\frac{1}{2}})dt = (\frac{3}{4}t^2 + \frac{2}{3}t^{\frac{3}{2}})\,|_0^9 = \frac{315}{4}$

10. $\int_1^x \frac{1}{t}\,dt = 1 \Rightarrow \ln x = 1 \Rightarrow x = e^1 = e.$

11. See graph for $r(t) = 2\sin(\frac{\pi}{4}t)$



(a) Level is highest when $t=4$ because the rate of change is positive before 4 and negative after 4.

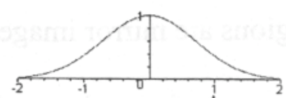
(b) $\int_0^4 2\sin(\frac{\pi}{4}t)dt = -2\cos(\frac{\pi}{4}t)\frac{4}{\pi}\,|_0^4 = \frac{8}{\pi} - (-\frac{8}{\pi}) = \frac{16}{\pi}$

Hence the highest water level is $30 + \frac{16}{\pi}$ gallons.

(c) The minimum level will be 30 gallons.

(d) Looking at the graph we can see that the signed area from 4 to 5 will be the same size, just negative, as the area from 0 to 1. Hence the amount lost from 4 to 5 is the same as the amount gained from 0 to 1.

12. See the graph of e^{-t^2} as aid for $g(x) = \int_0^x e^{-t^2}dt$.



(a) $g(x) = 0$ when $x = 0$.
$g(x) > 0$ when $x > 0$.
$g(x) < 0$ when $x < 0$.

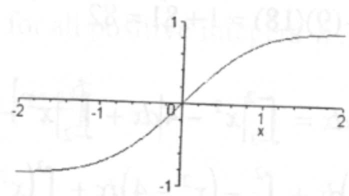
(When $x<0$, $g(x) = \int_0^x e^{-t^2}dt = -\int_x^0 e^{-t^2}dt = -$(positive area) = negative number)

(b) $g(x)$ is always increasing, never decreasing.

(c) $g'(x) = e^{-x^2} \Rightarrow g''(x) = -2xe^{-x^2}$. Hence
$g''(x) > 0$ if $x < 0$ so $g(x)$ is concave up on $(-\infty, 0)$, while
$g''(x) < 0$ if $x > 0$ so $g(x)$ is concave down on $(0, \infty)$.

(d) $g(-x) = \int_0^{-x} e^{-t^2}dt = -\int_{-x}^0 e^{-t^2}dt = -$(area of y reflection)$= -\int_0^x e^{-t^2}dt = -g(x)$,
hence $g(x)$ is an odd function.

(e) See graph to the right.



(f) $g(1) \cong 0.75$
$g(-1) \cong -0.75$
$g(2) \cong 0.88$

19. $\int_a^b [f'(x)g(x) + g'(x)f(x)]dx = f(x)g(x)\Big|_a^b$ (Product Rule)
$= f(b)g(b) - f(a)g(a) = 5.5(-5) - 4(0) = -27.5$