

24.2 The Average Value of a Function: An Application of the Definite Integral

$$3. \quad \frac{1}{\pi} \int_0^{\pi} \sin x dx = \frac{1}{\pi} (-\cos x) \Big|_0^{\pi} = \frac{1}{\pi} (-(-1) - (-1)) = \frac{2}{\pi}$$

$$4. \quad (a) \quad vel_{ave} = \frac{1}{6} \int_0^6 (-t^2 + 4t) dt = \frac{1}{6} \left(-\frac{1}{3}t^3 + 2t^2\right) \Big|_0^6 = \frac{1}{6} \left[-\frac{1}{3}(6)^3 + 2(6^2) - (0)\right] \\ = (-2)(6) + 2(6) = 0$$

$$(b) \quad speed_{ave} = \frac{1}{6} \int_0^6 |-t^2 + 4t| dt \\ = \frac{1}{6} \left[\int_0^4 (-t^2 + 4t) dt + \int_4^6 -(-t^2 + 4t) dt \right] = \frac{1}{6} \left[\left(-\frac{1}{3}t^3 + 2t^2\right) \Big|_0^4 + \left(\frac{1}{3}t^3 - 2t^2\right) \Big|_4^6 \right] \\ = \frac{1}{6} \left[\left(-\frac{1}{3}(4)^3 + 2(4^2) - (0)\right) + \left(\frac{1}{3}(6)^3 - 2(6^2) - \left(\frac{1}{3}(4)^3 - 2(4^2)\right)\right) \right] \\ = \frac{1}{6} \left[2\left(-\frac{1}{3}(4)^3 + 2(4^2)\right) \right] = \frac{1}{6} (2)(4^2) \left[-\frac{4}{3} + 2\right] = \frac{1}{3} (4^2) \left[\frac{2}{3}\right] = \frac{32}{9}$$

$$6. \quad \frac{1}{1} \int_0^1 5e^{-t} dt = -5e^{-t} \Big|_0^1 = -5e^{-1} - (-5e^0) = 5\left(1 - \frac{1}{e}\right) \text{ grams.}$$

$$7. \quad (a) \quad s(t) = |3 \sin(\pi t)|$$

$$(b) \quad \int_0^2 3 \sin(\pi t) dt = -\frac{3}{\pi} \cos(\pi t) \Big|_0^2 = -\frac{3}{\pi} \cos(2\pi) - \left(-\frac{3}{\pi} \cos(0\pi)\right) = 0$$

$$(c) \quad \int_0^2 |3 \sin(\pi t)| dt = \int_0^1 |3 \sin(\pi t)| dt + \int_1^2 |3 \sin(\pi t)| dt = \int_0^1 3 \sin(\pi t) dt + \int_1^2 -3 \sin(\pi t) dt \\ = -\frac{3}{\pi} \cos(\pi t) \Big|_0^1 + \frac{3}{\pi} \cos(\pi t) \Big|_1^2 = \left[-\frac{3}{\pi} \cos(1\pi) - \left(-\frac{3}{\pi} \cos(0\pi)\right)\right] + \left[\frac{3}{\pi} \cos(2\pi) - \frac{3}{\pi} \cos(1\pi)\right] \\ = \left[-\frac{3}{\pi}(-1) - \left(-\frac{3}{\pi}(1)\right)\right] + \left[\frac{3}{\pi}(1) - \frac{3}{\pi}(-1)\right] = 4\left(\frac{3}{\pi}\right) = \frac{12}{\pi}$$

$$(d) \quad \frac{1}{2} \int_0^2 3 \sin(\pi t) dt = 0$$

$$(e) \quad \frac{1}{2} \int_0^2 |3 \sin(\pi t)| dt = \frac{1}{2} \frac{12}{\pi} = \frac{6}{\pi}$$

8. III < I = IV < II (same order as size of area under the curve.)