

## 25.1 A List of Basic Antiderivatives

2.  $A \frac{1}{n+1} t^{n+1} + C$     6.  $\frac{5}{7} \ln|x| + C$     10.  $\frac{1}{5} \tan t + C$     16. (a)  $y' = -\frac{\pi}{3}(-\sin(3x)(3)) = \pi \sin(3x)$     (b)  $-\frac{A}{B} \cos(Bx) + C$
4.  $\frac{1}{2} \ln|x| + C$     8.  $\arctan x + C$     18. (a)  $\int \frac{2+x}{x} dx = \int (\frac{2}{x} + 1) dx = 2 \ln|x| + x + C$
- (b)  $\int \frac{3}{x^2} dx = \int 3x^{-2} dx = 3(-1)x^{-1} + C = -\frac{3}{x} + C$
- (c)  $3 \arctan x + C$
- (d)  $\int (\frac{t^3}{4} + \frac{4}{\sqrt{t}}) dt = \int (\frac{1}{4}t^3 + 4t^{-\frac{1}{2}}) dt = \frac{1}{4} \cdot \frac{1}{4} t^4 + 4(2)t^{\frac{1}{2}} + C = \frac{1}{16} t^4 + 8\sqrt{t} + C$

## 25.2 Substitution: The Chain Rule in Reverse

- (a) Use  $u = 2x^2 + 1$ ,  $du = 4x dx$ :  $\int \sqrt{u} \frac{1}{4} du = \frac{1}{4} \cdot \frac{2}{3} u^{\frac{3}{2}} + C = \frac{1}{6} (2x^2 + 1)^{\frac{3}{2}} + C$
- (b) Use  $u = 2x^2 + 1$ ,  $du = 4x dx$ :  $\int u^{-\frac{1}{2}} \cdot \frac{1}{4} du = \frac{1}{4} \cdot 2u^{\frac{1}{2}} + C = \frac{1}{2} \sqrt{2x^2 + 1} + C$
- (c) Use  $u = \sqrt{x}$ ,  $2du = \frac{1}{\sqrt{x}} dx$ :  $\int \cos(u) \cdot 2du = 2 \sin(u) + C = 2 \sin(\sqrt{x}) + C$
- (d) Use  $u = \cos x$ ,  $-du = \sin x dx$ :  $\int -\sqrt{u} du = -\frac{2}{3} u^{\frac{3}{2}} + C = -\frac{2}{3} (\cos x)^{\frac{3}{2}} + C$
5. (a)  $\frac{5}{3} \arctan(3x) + C$
- (b) Use  $u = \ln x$ ,  $du = \frac{1}{x} dx$ :  $\int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$
- (c)  $\int e^{2x} dx = \frac{1}{2} e^{2x} + C$
- (d) Use  $u = \ln w$ ,  $du = \frac{1}{w} dw$ :  $\int u^2 du = \frac{1}{3} u^3 + C = \frac{1}{3} (\ln w)^3 + C$
6. (a) Use  $u = t^3$ ,  $du = 3t^2 dt$ :  $\int \sin u \cdot \frac{1}{3} du = -\frac{1}{3} \cos u + C = -\frac{1}{3} \cos(t^3) + C$
- (b) Use  $u = -x^2$ ,  $du = -2x dx$ :  $\int e^u \cdot \frac{-1}{2} du = -\frac{1}{2} e^u + C = -\frac{1}{2} e^{-x^2} + C$
- (c)  $\ln|x+5| + C$
- (d) Use  $u = 2t^2 + 7$ ,  $du = 4t dt$ :  $\int \frac{1}{u} \cdot \frac{1}{4} du = \frac{1}{4} \ln|u| + C = \frac{1}{4} \ln|2t^2 + 7| + C$
9. (a)  $3 \int_0^1 \frac{1}{1+(2w)^2} dw = \frac{3}{2} \arctan(2w) \Big|_0^1 = \frac{3}{2} \arctan(2) - \frac{3}{2} \arctan(0) = \frac{3}{2} \arctan(2)$
- (b)  $\int_0^1 \frac{1}{3} + \frac{4}{3} w^2 dw = \left( \frac{1}{3} w + \frac{4}{9} w^3 \right) \Big|_0^1 = \frac{1}{3} + \frac{4}{9} = \frac{7}{9}$
- (c)  $\int_{\pi/2}^{3\pi/2} \cos\left(\frac{t}{2}\right) dt = 2 \sin\left(\frac{t}{2}\right) \Big|_{\pi/2}^{3\pi/2} = 2 \sin\left(\frac{3\pi}{4}\right) - 2 \sin\left(\frac{\pi}{4}\right) = 2 \cdot (-1) - 2 \cdot \frac{\sqrt{2}}{2} = -2 - \sqrt{2}$
- (d)  $\frac{4}{3} \ln|3x+2| \Big|_1^3 = \frac{4}{3} (\ln 11 - \ln 5) = \frac{4}{3} \ln\left(\frac{11}{5}\right)$
- (e) Use  $u = 2x+1$ ,  $du = 2dx$ :  $\int_3^9 u^{-2} \cdot \frac{1}{2} du = -\frac{1}{2} u^{-1} \Big|_3^9 = -\frac{1}{2} \left(\frac{1}{9} - \frac{1}{3}\right) = -\frac{1}{2} \cdot \frac{-2}{9} = \frac{1}{9}$