

25.(a) Substitute $u = \sin x$, $du = \cos x dx$: $\int \cos x \cdot \sin x dx = \int u du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2 x + C$

(b) Substitute $u = \cos x$, $du = -\sin x dx$: $\int \cos x \cdot \sin x dx = -\int u du = -\frac{1}{2}u^2 + D = -\frac{1}{2}\cos^2 x + D$

(c) The two anti derivatives are vertical shifts of each other since $\frac{1}{2}\sin^2 x - (-\frac{1}{2}\cos^2 x) = \frac{1}{2} \cdot 1$

25.3 Substitution to alter the Form of an Integral

2. Use $u = x - 7$, $du = dx$, $x = u + 7$:

$$\int \frac{u+10}{u} du = \int (1 + \frac{10}{u}) du = u + 10 \ln|u| + C = x - 7 + 10 \ln|x - 7| + C \quad \text{or: } x + 10 \ln|x - 7| + D$$

4. Use $u = 3 + x$, $du = dx$, $x = u - 3$:

$$\int \frac{2(u-3)}{u} du = \int (2 - \frac{6}{u}) du = 2u - 6 \ln|u| + C = 2(3 + x) - 6 \ln|3 + x| + C \quad \text{or: } 2x - 6 \ln|3 + x| + D$$

6. Use $u = 2t + 5$, $du = 2 dt$, $t = \frac{u-5}{2}$:

$$\int \frac{u-5}{2} \sqrt{u} \frac{1}{2} du = \int (\frac{1}{4}u^{\frac{3}{2}} - \frac{5}{4}u^{\frac{1}{2}}) du = \frac{1}{4} \cdot \frac{2}{5}u^{\frac{5}{2}} - \frac{5}{4} \cdot \frac{2}{3}u^{\frac{3}{2}} + C = \frac{1}{10}(2t+5)^{\frac{5}{2}} - \frac{5}{6}(2t+5)^{\frac{3}{2}} + C$$

8. Use $u = \sin(3x)$, $\frac{du}{dx} = 3 \cos(3x)$, $\frac{1}{3} du = \cos(3x) dx$:

$$\int \frac{\cos(3x)}{\sin(3x)} dx = \int \frac{1}{u} \cdot \frac{1}{3} du = \frac{1}{3} \ln|u| + C = \frac{1}{3} \ln|\sin(3x)| + C$$

10. Use $u = \sin t$, $du = \cos t$: $\int u^4 du = \frac{1}{5}u^5 + C = \frac{1}{5}(\sin t)^5 + C$