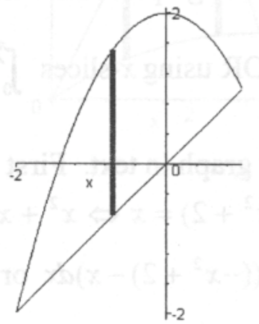
27.2 Slicing to Find the Area Between Two Curves

2. $y = 2 - x^2$ and $y = x$ intersect when

   $2 - x^2 = x \Rightarrow x^2 + x - 2 = 0 \Rightarrow x = -2, 1$.

   Area $= \int_{-2}^{1} ((2 - x^2) - x)dx = \left(2x - \frac{1}{3}x^3 - \frac{1}{2}x^2\right)\Big|_{-2}^{1} = \frac{9}{2}$

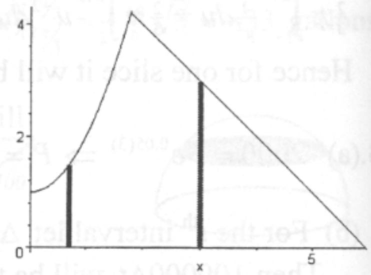


4.

$$-x + 6 = x^2 + 1 \Rightarrow x^2 + x - 5 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(-5)}}{2} = \frac{-1 \pm \sqrt{21}}{2}$$

$$\text{Area} = \int_0^{\frac{-1+\sqrt{21}}{2}} (x^2 + 1)dx + \int_{\frac{-1+\sqrt{21}}{2}}^{6} (-x + 6)dx =$$

$$\left(\tfrac{1}{3}x^3 + x\right)\Big|_0^{\frac{-1+\sqrt{21}}{2}} + \left(-\tfrac{1}{2}x^2 + 6x\right)\Big|_{\frac{-1+\sqrt{21}}{2}}^{6} \approx 3.707 + 8.857 = 12.564$$



7. See the graph in the text. $y = \tan x \Leftrightarrow x = \arctan y$. Then using y-slices we get $\int_0^1 (\pi/2 - \arctan y)dy$

   OR using x-slices $\int_0^{\pi/4} \tan x\, dx + \int_{\pi/4}^{\pi/2} 1\, dx$

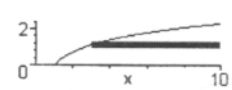
8. See graph in text. First we find the intersection points.

   $(-x^2 + 2) = x \Leftrightarrow x^2 + x - 2 = 0 \Rightarrow x = -2, 1$. Also $y = -x^2 + 2 \Rightarrow x^2 = -y + 2 \Rightarrow x = \pm\sqrt{-y + 2}$.

   $\int_{-2}^{1} ((-x^2 + 2) - x)dx$ or $\int_{-2}^{1} (y - (-\sqrt{-y+2}))dy + \int_1^2 (\sqrt{-y+2} - (-\sqrt{-y+2}))dy$

10. $y = \ln x \Leftrightarrow x = e^y$. Using y-slices the area is

    $\int_0^{\ln 10} (10 - e^y)dy = \left(10y - e^y\right)\Big|_0^{\ln 10} = (10\ln 10 - 10) - (0 - 1) = 10\ln(10) - 9$



12. Line is $(y - 1) = \frac{(1-0)}{(e-2e)}(x - e) \Leftrightarrow x = -e(y - 1) + e \Rightarrow x = -ey + 2e$

    $\int_0^1 ((-ey + 2e) - e^y)dy = \left(-\frac{e}{2}y^2 + 2ey - e^y\right)\Big|_0^1 = -\frac{e}{2} + 2e - e - (-1) = \frac{e}{2} + 1$

    OR $\int_1^e \ln x\, dx + \int_e^{2e} (-\frac{1}{e}x + 2)dx = \frac{e}{2} + 1$