ar's Rule

Appendix F

1. Use L'Hopital's rule since it is of the form $\frac{\infty}{\infty}$: $\lim_{x\to\infty}\frac{\ln x}{x} = \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = 0$.

2. $\lim_{x\to 0^+}\frac{\ln x}{x} = -\infty$ since it is of the form $\frac{-\infty}{0^+}$.

5. Use L'Hopital's rule three times: $\lim_{x\to\infty}\frac{100x^3}{e^x} = \lim_{x\to\infty}\frac{300x^2}{e^x} = \lim_{x\to\infty}\frac{600x}{e^x} = \lim_{x\to\infty}\frac{600}{e^x} = 0$.

12. $\lim_{x\to 0.5}\frac{\ln(1-2x)}{2x-1}$ is undefined. Note: $\lim_{x\to 0.5^+}\frac{\ln(1-2x)}{2x-1} = -\infty$ since it is of type $\frac{-\infty}{0^+}$

13. Use L'Hopital's rule and simplify : $\lim_{x\to\infty}\frac{e^{2x}+7}{5e^{3x}-10} = \lim_{x\to\infty}\frac{2e^{2x}}{15e^{3x}} = \lim_{x\to\infty}\frac{2}{15e^{x}} = 0$

18. Use L'Hopital's rule: $\lim_{x\to\infty}\frac{x^3+10000x}{3^x} = \lim_{x\to\infty}\frac{3x^2+10000}{\ln 3\cdot 3^x} = \lim_{x\to\infty}\frac{6x}{(\ln 3)^2 3^x} = \lim_{x\to\infty}\frac{6}{(\ln 3)^3 3^x} = 0$.

ppendix G

2. $f(x) = e^x - x - 3$, $f'(x) = e^x - 1$, if $x_0 = 2$ (other first guesses are possible) t

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \approx 2 - \frac{2.3890560989}{6.3890560989} \approx 1.626070571$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \approx 1.626070571 - \frac{.457788186}{4.083858757} \approx 1.513973603$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx 1.513973603 - \frac{.030780407}{3.544754010} \approx 1.505290234$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \approx 1.505290234 - \frac{.000170845}{3.505461079} \approx 1.505241497$

The last two agree to three decimal places, so we can stop.

4. $f(x) = x^3 - 20$, $f'(x) = 3x^2$, if $x_0 = 2.5$ (other first guesses are possible) then

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.5 - \frac{-4.375}{18.75} \approx 2.733333333$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \approx 2.733333333 - \frac{.42103703}{22.4133333} \approx 2.714548219$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx 2.714548219 - \frac{.00288700}{22.10631610} \approx 2.714417623$

The last two agree to three decimal places, so we can stop.

8. $f(x) = \cos x - x$, $f'(x) = -\sin x - 1$, if $x_0 = \frac{1}{2}$ (other first guesses are possible) then

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \approx .5 - \frac{.3775825619}{-1.479425539} \approx .7552224170$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \approx .7552224170 - \frac{-.0271033117}{-1.685450632} \approx .7391416661$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx .7391416661 - \frac{-.0000946153}{-1.673653811} \approx .7390851339$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \approx .7390851339 - \frac{.0000000011}{-1.673612030} \approx .7390851332$

The last two agree to five decimal places, so we can stop.