Math Xb
Worksheet—Related Rates—Solution

Spring 2004

A water tank has the shape of an inverted circular cone with base radius of 6 feet and height of 12 feet. If water is being pumped into the tank at a rate of 10 cubic feet per minute, find the rate at which the water level is rising when the water is 5 feet deep.

Solution. Let V, r, and h be the volume of the water, the radius of the cone, and the height of the cone at time t, where t is measured in minutes. We are given that $dV/dt = 10\text{ft}^3/\text{min}$ and are asked to find dh/dt when h is 5 feet. The quantities V, r, and h are related by the volume of a cone,

$$V = \frac{1}{3}\pi r^2 h.$$

It would be very useful to express the volume of the cone as a function of h alone. In order to eliminate r, we can use similar triangles to write

$$\frac{r}{h} = \frac{6}{12}$$

or $r = h/2$. The expression for V now becomes

$$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3.$$

Now we can differentiate each side with respect to t,

$$\frac{dV}{dt} = \frac{\pi}{12}\left(3h^2\frac{dh}{dt}\right) = \frac{\pi}{4}h^2\frac{dh}{dt},$$

or

$$\frac{dh}{dt} = \frac{4}{\pi h^2}\frac{dV}{dt}.$$

Substituting $h = 5$ and $dV/dt = 10$, we obtain

$$\frac{dh}{dt} = \frac{4}{\pi 5^2} \cdot 10 = \frac{8}{5}\pi.$$

Thus, the water level is rising at a rate of $8\pi/5 \approx 5.027$ feet per minute.