Math Xb Spring 2004
Worksheet: Solving Trigonometric Equations
March 8, 2004

1. Simplify each of the following to an expression that does not involve trig functions or inverse trig functions.

(a) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\pi/4$

(b) $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$

(c) $\cos\left(\cos^{-1}.7\right) = .7$

(d) $\tan^{-1}(\tan\pi) = 0$

$\tan\pi = 0, \quad \tan^{-1}(0) = 0$

(e) $\tan\left(\sin^{-1}(a)\right) = \frac{a}{\sqrt{1-a^2}}$

(f) $\tan\left(\sin^{-1}(-a)\right) = \frac{-a}{\sqrt{1-a^2}}$

2. Many animal populations, such as that of rabbits, fluctuate over ten-year cycles. Suppose that the number of rabbits at time t (in years) is given by

$$N(t) = 1000\cos\left(\frac{\pi}{5}t\right) + 4000.$$

For what values of t between 0 and 10 does the rabbit population equal 4500?

$4500 = 1000\cos\left(\frac{\pi}{5}t\right) + 4000$

$500 = 1000\cos\left(\frac{\pi}{5}t\right)$

$.5 = \cos(\pi/5\, t)$

$\text{period} = \frac{2\pi}{B} = \frac{2\pi}{\pi/5} = 10$

$\cos^{-1}(.5) = \frac{\pi}{3} + 2\pi n, \quad \frac{5\pi}{3} + 2\pi n$

$\frac{\cos^{-1}(.5)}{(\pi/5)} = t \rightarrow t = \frac{\pi/3}{\pi/5}, \frac{7\pi/3}{\pi/5}, \frac{5\pi/3}{\pi/5}$ >10

3. For each of the following equations, find all solutions in the interval specified. Give exact answers like $\frac{\pi}{4}$ as opposed to numerical approximations like 0.7854.

(a) $2\cos^2 t + 3\cos t + 1 = 0$ for $t \in [0, 2\pi]$

$X = \cos t$

$2X^2 + 3X + 1 = 0 \Rightarrow X = \frac{-3 \pm \sqrt{9-8}}{4} = \frac{-3 \pm 1}{4} = -\frac{1}{2}, -1$

$\cos t = -\frac{1}{2} \Rightarrow \boxed{t = \frac{2\pi}{3}, \frac{4\pi}{3}}$

(b) $\sin x - \cos x = 0$ for $x \in [0, 2\pi]$

$\sin X = \cos X \Rightarrow \boxed{X = \pi/4, 5\pi/4}$

$\tan X = 1$

(c) $\cos\left(2x - \frac{\pi}{4}\right) = 0$ for $x \in [-2\pi, 2\pi]$

$2X - \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$ $\quad \left(\text{period} = \frac{2\pi}{2} = \pi\right)$

$2X = \frac{3\pi}{4}, \frac{7\pi}{4}, -\frac{\pi}{4}, -\frac{5\pi}{4} \Rightarrow X = \frac{3\pi}{8}, \frac{7\pi}{8}, -\frac{\pi}{8}, -\frac{5\pi}{8}$

$\frac{11\pi}{8}, \frac{15\pi}{8}, -\frac{9\pi}{8}, -\frac{13\pi}{8}$

(d) $4\sin^2 x \tan x = \tan x$ for $x \in (-\infty, \infty)$

Subtract / Factor

$\tan X(4\sin^2 X - 1) = 0$

$\tan X = 0$ or $4\sin^2 X = 1 \Rightarrow \sin X = \pm \frac{1}{2}$

$X = \pm\frac{\pi}{6} + 2n\pi, \pm\frac{5\pi}{6} + 2n\pi, \quad n \in \mathbb{Z}.$

or $X = n\pi, \quad n \in \mathbb{Z}$