SOLUTIONS

Math Xb Spring 2004
Worksheet: The Substitution Rule
April 21, 2004

1. Evaluate the following integrals.

(a) $\int (6x^2+2)\sin(x^3+x+1)\,dx$ $= \int \sin u \cdot 2\,du = 2\int \sin u\,du$

$u = x^3+x+1$

$du = (3x^2+1)\,dx$

$2\,du = (6x^2+2)\,dx$

$= -2\cos u + C$

$= \boxed{-2\cos(x^3+x+1)+C}$

(b) $\cancel{\int (1+x^2)^{3/2} x^3\,dx}$ $\to \int (1+x^3)^{3/2} x^2\,dx$

$u = 1+x^3$

$du = 3x^2\,dx$

$\frac{1}{3}\,du = x^2\,dx$

$= \int u^{3/2}\cdot\frac{1}{3}\,du = \frac{1}{3}\int u^{3/2}\,du = \frac{1}{3}\frac{u^{5/2}}{5/2}+C$

$= \boxed{\frac{2}{15}\left((1+x^3)\right)^{5/2}+C}$

(c) $\int_0^4 \sqrt{2x+1}\,dx$

$u = 2x+1$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$= \int_1^9 \sqrt{u}\cdot\frac{1}{2}\,du$

$= \frac{1}{2}\int_1^9 u^{1/2}\,du = \frac{1}{2}\cdot\frac{u^{3/2}}{3/2}\Big|_1^9 = \frac{1}{3}u\sqrt{u}\Big|_1^9$

$= \frac{1}{3}(9\cdot 3 - 1\cdot 1) = \boxed{26/3}$

(d) $\int_{-\pi}^{\pi} x^2\sin 7x\,dx$

odd function, so $\int = \boxed{0}$.

(e) $\int x^2 e^{x^3}\,dx$ $= \int \frac{1}{3}e^u\,du = \frac{1}{3}e^u + C$

$u = x^3$

$du = 3x^2\,dx$

$\frac{1}{3}\,du = x^2\,dx$

$= \boxed{\frac{1}{3}e^{x^3}+C}$

(f) $\int \frac{\tan^{-1}x}{1+x^2}\,dx$ $= \int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(\tan^{-1}x)^2+C}$

$u = \tan^{-1}x$

$du = \frac{1}{1+x^2}\,dx$

(g) $\int_0^{\pi/4}\tan x\,dx$ $= \int_0^{\pi/4}\frac{\sin x}{\cos x}\,dx = -\int_1^{1/\sqrt{2}}\frac{1}{u}\,du = -\ln|u|\Big|_1^{1/\sqrt{2}}$

$u = \cos x$

$du = -\sin x\,dx$

$= -\ln(1/\sqrt{2}) + \ln 1$

$= -\ln 1 + \ln\sqrt{2} + \ln 1$

$= \ln\sqrt{2} = \boxed{\frac{\ln 2}{2}}$

2. Suppose that $\int_0^{12} g(x)\,dx = \frac{\pi}{12}$. Evaluate $\int_0^3 g(4x)\,dx$.

$u = 4x$

$du = 4\,dx$

$\frac{1}{4}\,du = dx$

$\int_0^3 g(4x)\,dx = \int_0^{12}\frac{1}{4}g(u)\,du$

$= \frac{1}{4}\int_0^{12} g(u)\,du = \frac{1}{4}\cdot\frac{\pi}{12} = \boxed{\frac{\pi}{48}}$