

Math Xb Spring 2004
Worksheet: Summation Notation
February 25, 2004

1. Express the following sums and series using summation notation.

(a) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$

$$= \sum_{i=1}^5 \frac{1}{2^i}$$

(b) $4 + \frac{12}{5} + \frac{36}{25} + \cdots + \frac{4 \cdot 3^8}{5^8}$

$$= \sum_{i=0}^8 \frac{4 \cdot 3^i}{5^i}$$

(c) $7 - 14 + 28 - 56 + \cdots$

$$= \sum_{i=0}^{\infty} (-1)^i \cdot 7 \cdot 2^i$$

2. For each of the following series, determine if the series converges or diverges. If it converges, determine its sum.

(a) $\sum_{n=0}^{\infty} \frac{3}{2} \left(\frac{5}{2}\right)^n$ diverges

(b) $\sum_{n=0}^{\infty} \frac{15}{100} \left(\frac{1}{10}\right)^n$ converges

$$S = \frac{a}{1-r} = \frac{15/100}{1-1/10} = \frac{15}{100 \cdot \frac{9}{10}} = \frac{15}{90} = \frac{1}{6}$$

(c) $\sum_{n=1}^{\infty} \frac{3}{2} \left(-\frac{2}{3}\right)^n$ converges

$$S = \frac{a}{1-r} = \frac{-1}{1+2/3} = \frac{-1}{5/3} = -3/5$$

3. Determine whether each of the following statements is always true or not always true.

(a) $\sum_{i=1}^n c = nc$ true

(b) $\sum_{i=1}^n (a_i + b_i) = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i$ true

(c) $\sum_{i=1}^n a_i b_i = \left(\sum_{i=1}^n a_i\right) \left(\sum_{i=1}^n b_i\right)$ not always true

4. Which of the following are equal to $\sum_{i=1}^{10} 5 \cdot 2^i$? $= \frac{10 - 2^{11}}{-1} = 2^{11} - 10$

(a) $\sum_{i=0}^9 5 \cdot 2^i$ no

(b) $\sum_{i=0}^9 5 \cdot 2^{i+1}$ yes

(c) $\sum_{i=1}^{2^{10}} 5i$ no

(d) $\sum_{k=4}^{13} 5 \cdot 2^{k-3}$ yes

(e) $5 \cdot 2^{11} - 10$ yes