Math Xb Spring 2004
Worksheet: Trigonometric Identities
March 10, 2004

1. Use the angle-addition formulas and the symmetry properties of sine and cosine to verify the following angle subtraction formula.

$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$

$\sin(x-y) = \sin(x+(-y)) = \sin x \cos(-y) + \sin(-y)\cos x$

$= \sin x \cos y - \sin y \cos x$

2. Use the angle-addition formulas to verify the following double-angle formula.

$$\sin 2x = 2\sin x \cos x$$

$\sin(2x) = \sin(x+x) = \sin x \cos x + \sin x \cos x = 2\sin x \cos x$

3. Use the double-angle formulas for cosine to verify the following power-reducing formula.

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$\frac{1}{2}(1-\cos 2x) = \frac{1}{2}(1-(1-2\sin^2 x)) = \frac{1}{2} - \frac{1}{2} + \sin^2 x = \sin^2 x$

4. Use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ to verify the following Pythagorean identity.

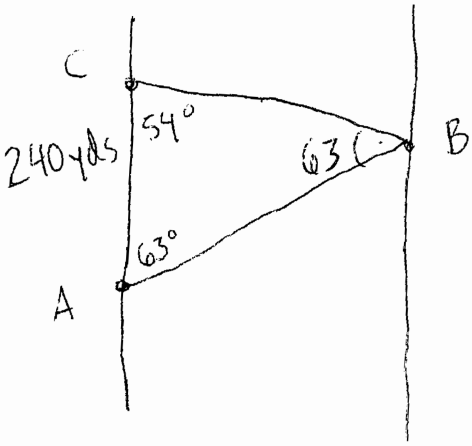
$$1 + \cot^2 x = \csc^2 x$$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \left(\frac{1}{\sin^2 x}\right)(\sin^2 x + \cos^2 x) = \frac{1}{\sin^2 x} \Rightarrow$

$1 + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \Rightarrow 1 + \cot^2 x = \csc^2 x$

5. To find the distance between two points A and B that lie on opposite banks of a river, a surveyor selects a third point C on the same bank of the river as A. The surveyor notes that the distance between A and C is 240 yards, the angle $\angle BAC$ measures 63°, and the angle $\angle ACB$ measures 54°. Approximate the distance between A and B.

law of sine / cosine

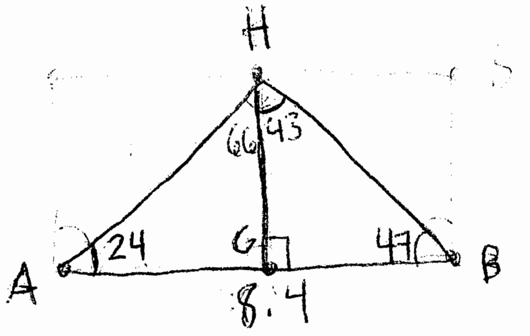


$$\frac{\sin(63)}{240} = \frac{\sin(54)}{AB}$$

$$\boxed{AB = 218 \text{ yds}}$$

6. The angles of elevation of a hot-air balloon from two points A and B on level ground are 24° and 47°, respectively. Points A and B are 8.4 miles apart and the balloon is between the two points, in the same vertical plane. Approximate the height of the balloon above the ground.

law of sines



$$\frac{8.4}{\sin(109)} = \frac{HB}{\sin(90)}$$

$$HB \approx 9$$

then $\dfrac{HG}{\sin 47} = \dfrac{9}{\sin 90} \Rightarrow HG = \boxed{6.6 \text{ miles}}$