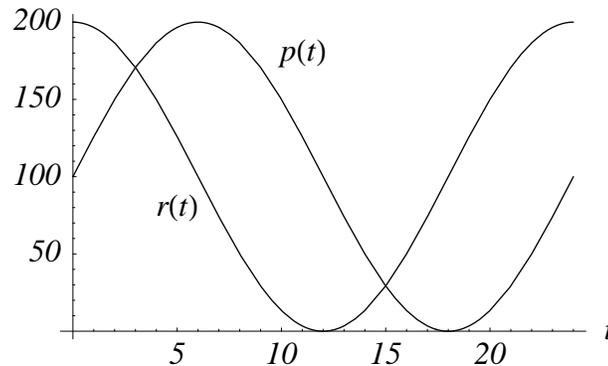


Math Xb Spring 2004  
Worksheet: The Definite Integral  
April 9, 2004

1. Suppose that the level of a certain hormone in the blood is determined by two separate processes: a hormone production process (perhaps the secretion rate of specialized cells that produce the hormone) and a hormone removal process (perhaps its deactivation by enzymes secreted by other cells).

Assume that the production rate  $p(t)$  and the removal rate  $r(t)$  depend on the time of day in some cyclic pattern that repeats every 24 hours or so. The graphs of  $p(t)$  and  $r(t)$  are given below, where  $t$  represents hours after midnight,  $p(t)$  is given in milligrams per hour produced, and  $r(t)$  is given in milligrams per hour removed.



- (a) When is the production rate  $p(t)$  at its highest?
- (b) When is the removal rate  $r(t)$  at its lowest?
- (c) Express as a definite integral (or sum or difference of definite integrals) the net change in hormone level between 5 a.m. and 8 p.m.
- (d) At approximately what time is the hormone level in the blood highest?
- (e) At approximately what time is the hormone level in the blood lowest?
2. After World War II, the birth rate in western countries increased dramatically. Suppose that the number of babies born (in millions per year) was given by  $b(t) = 5 + 2t$  for  $0 \leq t \leq 10$ , where  $t$  is time in years after the end of the war.
- (a) Express as a definite integral the total number of babies born during the first ten years after the war. Then evaluate this definite integral.
- (b) Express the following statement as an equation involving an appropriate definite integral: "The total number of babies born from the end of the war up to the time  $T$  was precisely 14 million." Then find  $T$ .
3. Evaluate the following definite integrals by interpreting each as a signed area.

(a)  $\int_0^{4\pi} \sin x \, dx$

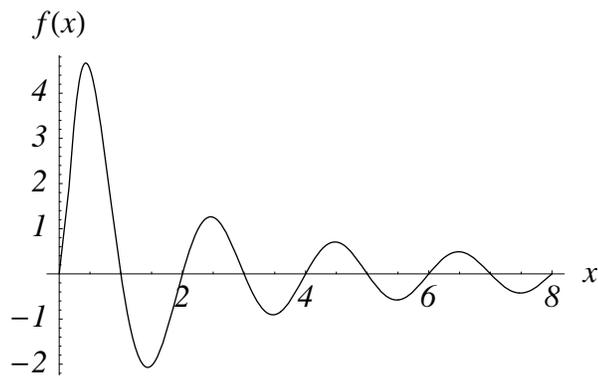
(b)  $\int_{-a}^a x^3 \, dx$

(c)  $\int_{-5}^5 \frac{x^2 - 2x^4}{x^3 + x} \, dx$

(d)  $\int_0^{10} |x - 4| dx$

(e)  $\int_0^{10} (|x| - 4) dx$

4. Let  $f(x) = \frac{10\pi x \sin(\pi x)}{1 + (\pi x)^2}$ , as shown in the graph below.



Put the following integrals in ascending order, from most negative to most positive.

(a)  $\int_2^8 f(x) dx$

(b)  $\int_1^2 f(x) dx$

(c)  $\int_2^2 f(x) dx$

(d)  $\int_0^1 f(x) dx$

(e)  $\int_5^6 f(x) dx$

(f)  $\int_2^6 f(x) dx$