Math Xb Spring 2004
Worksheet: Modeling with Differential Equations
May 5, 2004

1. A population of otters is declining. The birth rate of the otters is b% per year, but the death rate of the otters is d% per year, where $d > b$. A group of people wants to prevent the otter population from dying out, so they plan to bring in otters from elsewhere at a constant rate of N otters per year. Let $P(t)$ be the population t years from now. We will model the situation with continuous functions.
   (a) Write a differential equation whose solution is $P(t)$.
   (b) What can we say about the value of N if we want to maintain a constant population of otters?
   (c) Suppose that $b = 4$, $d = 9$, $N = 40$, and the initial population of the otters is 1000. Solve the differential equation from part (a) using the technique discussed in yesterday's lab.
   (d) Why is it important for the problem to state that the situation will be modeled with continuous functions?

2. In a laboratory, a colony of fruit flies are under scrutiny. Let $P = P(t)$ be the number of fruit flies in the colony at time t. Given ample food, the population would grow at a rate proportional to itself, but flies are continually being siphoned off to another lab at a constant rate of C flies per day.
   (a) Write a differential equation whose solution is $P(t)$.
   (b) One of your classmates is convinced that the solution to the differential equation in part (a) is
   $$P(t) = P_0e^{kt} - Ct.$$
   Show him algebraically that this is not a solution to the differential equation.
   (c) Your classmate is having a hard time giving up the solution he brought up in part (b). He sees that it does not satisfy the differential equation, but he still has a strong gut feeling that it ought to be right. Convince him that it is wrong by using a more intuitive argument. Use words and talk about fruit flies.

3. Let $M = M(t)$ be the amount of money in a bank account at time t, given in years. Suppose that
   $$\frac{dM}{dt} = 0.03M - 3000.$$
   (a) What scenario could be modeled by this differential equation?
   (b) Suppose the initial deposit is \$40,000. Will the account ever be depleted?
   (c) What is the minimum initial deposit that ensures the account will not be depleted?
   (d) Suppose instead that
   $$\frac{dM}{dt} = 0.03M - 3000 + 100t.$$
   What scenario could be modeled by this differential equation?

4. Suppose that the population $P(t)$ of lions in a savannah is given by the differential equation
   $$\frac{dP}{dt} = 0.03P - 0.0001P^2.$$
   (This population growth model is known as a *logistic growth model*. Such models are typically more valid than exponential growth models.)
   (a) Without the $-0.0001P^2$ term, we would have the familiar exponential growth differential equation. For what might the $-0.0001P^2$ term be accounting?
   (b) What population values result in a constant population? That is, for what values of P is the rate of change in population zero?