Math Xb Spring 2004
Worksheet: Properties of the Definite Integral
April 12, 2004

1. Show why each of the following properties of the definite integral is true, first in terms of net change in amount (use a velocity-position example) and then in terms of area under a curve.

   (a) If k is a constant, then $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$.

   (b) If $f(x) \le g(x)$ for all x in the interval $[a, b]$, then $\displaystyle\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$.

   (c) $\displaystyle\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

(d) If $m \leq f(x) \leq M$ for all x in the interval $[a, b]$, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$

2. Show why each of the following properties of the definite integral is true in terms of area under a curve.

(a) If f is an odd function, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.

(b) If f is an even function, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{-a}^{a} f(x)\,dx$.