

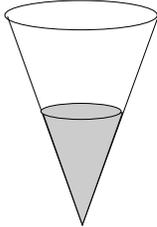
**Math Xb Spring 2005**

**Related Rates: Solutions to water tank problem**

**February 14, 2005**

A water tank has the shape of an inverted circular cone with base radius 6 feet and height 12 feet. If water is being pumped into the tank at a rate of 10 cubic feet per minute, find the rate at which the water level is rising when the water is 5 feet deep.

1. The figure below shows the cone.



2. The volume of water in the tank is represented by  $V$ , the height of the water by  $h$ , and the radius of the water by  $r$ .
3. We know that  $V'(t) = 10$  cubic feet per minute. We want to know  $h'(t)$ , the rate at which the water level (that is the height of the water) is rising.
4. Since the water is always in a cone shape, we can use the formula for the volume of a cone

$$V = \frac{1}{3}\pi r^2 h.$$

5. We differentiate both sides of the above equation. Note that the volume, height, and radius all change with time, so they are all functions of  $t$ . On the right hand side of the equation, we use the product rule.

$$V'(t) = \frac{\pi}{3} \cdot \left( 2r(t)r'(t)h(t) + (r(t))^2 h'(t) \right) \quad (\dagger)$$

6. We know that  $V'(t) = 10$ , and that  $h = 5$ , but we don't know  $r(t)$  or  $r'(t)$ , so we will need to find those. We use similar triangles (see figure) to see that

$$\begin{aligned} \frac{6}{12} &= \frac{r}{h} \\ r &= \frac{h}{2} \end{aligned}$$

So when  $h = 5$ ,  $r = \frac{5}{2}$ . Differentiating both sides of the above equation gives that  $r'(t) = \frac{h'(t)}{2}$ . Now we can substitute all of this information into equation ( $\dagger$ ) to get

$$10 = \frac{\pi}{3} \cdot \left( 2 \frac{5}{2} \frac{h'(t)}{2} 5 + \left( \frac{5}{2} \right)^2 h'(t) \right)$$

7. Solving for  $h'(t)$  gives

$$h'(t) = \frac{8}{5\pi}$$

8. The height is changing at a rate of  $\frac{8}{5\pi} \approx .5$  feet per minute. This is positive, so the water level is rising as stated in the problem, and a half a foot per minute seems like a fairly reasonable rate.