

Math Xb Spring 2005
Worksheet: L'Hôpital's Rule Day One
March 18, 2005

1. Evaluate each of the following limits.

(a) $\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + 3x + 2}$

(b) $\lim_{x \rightarrow 1} \frac{x - 1}{x^2 + 2x + 3}$

(c) $\lim_{x \rightarrow 0} \frac{x + \tan x}{\sin x}$

(d) $\lim_{x \rightarrow \infty} \frac{\ln(\ln x)}{x}$

(e) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x}$

(f) $\lim_{x \rightarrow \infty} \frac{\ln(x + 3)}{3}$

(g) $\lim_{x \rightarrow \infty} \frac{(\ln x)^3}{x^2}$

(h) $\lim_{x \rightarrow 0} \frac{1 - e^{-2x}}{\sec x}$

(i) $\lim_{x \rightarrow \infty} \frac{e^x}{x^n}$, where n is a positive integer

2. Given that

$$\begin{array}{lll} \lim_{x \rightarrow a} f(x) = 0 & \lim_{x \rightarrow a} g(x) = 0 & \lim_{x \rightarrow a} h(x) = 1 \\ & \lim_{x \rightarrow a} p(x) = \infty & \lim_{x \rightarrow a} q(x) = \infty \end{array}$$

which of the following are indeterminate forms? For those that are not an indeterminate form, evaluate the limit where possible.

(a) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$

(b) $\lim_{x \rightarrow a} \frac{f(x)}{p(x)}$

(c) $\lim_{x \rightarrow a} \frac{h(x)}{p(x)}$

(d) $\lim_{x \rightarrow a} \frac{p(x)}{f(x)}$

(e) $\lim_{x \rightarrow a} \frac{p(x)}{q(x)}$

3. (a) Find $\lim_{x \rightarrow \infty} \frac{e^x}{P(x)}$, where $P(x)$ is a polynomial. (Hint: Write $P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$. You'll have to use L'Hôpital's Rule more than once.)

- (b) What does this tell you about the rate of growth of e^x compared to the rate of growth of a polynomial?

4. (a) Find $\lim_{x \rightarrow \infty} \frac{\ln x}{x^p}$, where $p > 0$.

- (b) What does your answer to part (a) say about the relative rates of growth of $\ln x$ and any power of x as x grows very large?