Math Xb Spring 2005: Logarithmic Differentiation

February 9, 2005

1 Goals

- To use logarithmic differentiation to find the derivatives of functions of the form $f(x)^{g(x)}$
- To recognize when to use logarithmic differentiation

2 New Terms

- Logarithmic differentiation

3 Review

Review of procedure for finding the derivative of x^x from last time (last time there were two procedures, we'll generalize the second).

$$\begin{aligned} f(x) &= x^x \\ \ln(f(x)) &= \ln(x^x) \\ \ln(f(x)) &= x \ln x \\ \frac{f'(x)}{f(x)} &= 1 + \ln x \\ f'(x) &= x^x(1 + \ln x) \end{aligned}$$

4 Logarithmic Differentiation: Procedure

The steps of *logarithmic differentiation* (found on on page 538—539 of the textbook):

1. To use logarithmic differentiation, you need an equation $y = f(x)$, where $f(x) > 0$. Take the natural log of both sides.
2. Use log rules to bring down exponents and simplify expressions.
3. Differentiate both sides of the equation. Remember that y is a function of x, so
$$\frac{d}{dx}y = \frac{1}{y}\frac{dy}{dx} = \frac{y'}{y}$$
4. Solve for $\frac{dy}{dx}$ (or $f'(x)$).
5. Finally, to express y' in terms of x, replace y or $f(x)$ by the original function in terms of x.

5 Logarithmic Differentiation: When to Use It

1. Logarithmic differentiation is useful in differentiating a function that has a variable both in the base and the exponent.
2. Logarithmic differentiation can also be used to simplify the differentiation of a complex quotient. (There is an example of this in the book)

6 References

Section 17.2 in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*