# Math Xb Spring 2005
# Implicit differentiation

February 11, 2004

## 1 Goals

- To understand that an equation involving two variables describes a relationship between the two variables
- To use implicit differentiation to find $\frac{dy}{dx}$ given an equation involving $x$ and $y$
- To find the slope of a tangent line to a given curve (described by an equation involving $x$ and $y$) at a given point
- To find the points on a curve (described by an equation involving $x$ and $y$) at which the tangent line has a given slope

## 2 New Terms

- Implicit function
- Implicit differentiation

## 3 Implicit Functions

1. Note that so far, we have only found derivatives of functions of the form $y = f(x)$, where one variable is expressed *explicitly* as a function of the other. Since $y = x^2$ has this form, we can easily find $\frac{dy}{dx}$.
2. The equation $x = y^2$ does not have this form and its graph is not a function of $x$. However, *locally* the graph is a function except when $x = 0$.
3. We can find the slopes of the tangent lines to $x = y^2$ at the points $(5, \sqrt{5})$ and $(5, -\sqrt{5})$ by treating the curve as two functions, $y = \sqrt{x}$ and $y = -\sqrt{x}$.
4. We can also find $\frac{dy}{dx}$ by a technique called *implicit differentiation*.

## 4 Implicit Differentiation

1. In the previous example, we can also find $\frac{dy}{dx}$ by a technique called *implicit differentiation*. When an equation relating $x$ and $y$ does not define $y$ explicitly as a function of $x$, we can use *implicit differentiation* to find $\frac{dy}{dx}$.
2. Note that the formula for $\frac{dy}{dx}$ can include both $x$ and $y$. Since the original equation was not a function of $x$, a point on the curve must be specified by both an $x$- and $y$-value. Likewise, to find the slope of the tangent line at a point, we often need both the $x$- and $y$-value of the point.
3. Note that we used implicit differentiation in the last class when performing logarithmic differentiation.

## 5 Summary of Implicit Differentiation

1. Start with an equation relating $x$ and $y$.
2. Differentiate both sides of the equation with respect to the variable $x$, treating $y$ as a function of $x$ and using the Chain Rule as needed.
3. Solve the resulting equation for $\frac{dy}{dx}$.

## 6 References

Section 17.3 in *Calculus: An Integrated Approach to Functions and Their Rates of Change*.