Math Xb Spring 2005
Taylor Polynomials

1 Goals

- Be able to compute low-degree Taylor polys at any point.
- Understand that a Taylor poly gives a local approximation for a function (so they should also understand what it means to be a local approximation).
- Have a sense that the accuracy of the approximation goes up as the degree of the Taylor polynomial increases.
- Be able to use Taylor polynomials to find approximations (e.g. "approximate $e^{0.01}$ using a 2nd degree Taylor polynomial")

2 New Terms

- Taylor polynomial

A Taylor polynomial of degree n for a function $f(t)$ at a point $t = 0$ is is a polynomial $P_n(t)$ of degree n such that

$$\begin{aligned} P_n(0) &= f(0) \\ P_n'(0) &= f'(0) \\ P_n''(0) &= f''(0) \\ &\vdots \\ P_n^{(n)} &= f^{(n)}(0). \end{aligned}$$

Remember that $f^{(n)}(t)$ is the nth derivative of the function $f(t)$. In words, we say that the value of the polynomial is the same as the value of the function at zero, and that the first n derivatives are also the same at zero.

Show that the P_0, P_1, and P_3 that we have found are the first, second, and third degree Taylor polynomials.

3 General Formula (3 minutes)

The general formula for computing of Taylor polynomials follows (also in the textbook the book also p. 929).

The nth degree Taylor polynomial generated by $f(x)$ at $x = b$ is given by

$$P_n(x) = f(b) + \frac{f'(b)}{1!}(x-b) + \frac{f''(b)}{2!}(x-b)^2 + \frac{f'''(b)}{3!}(x-b)^3 + \cdots + \frac{f^{(n)}(b)}{n!}(x-b)^n$$

4 References

Section 30.1 in *Calculus: An Integrated Approach to Functions and Their Rates of Change*.