

Math Xb Spring 2005

Graphs of Sine and Cosine Functions

(Corrected)

February 23, 2005

1 Goals

- Identify the balance value, amplitude, and period of a sinusoidal function given its formula or graph
- Be able to write the formula for a sinusoidal function given information about the period, amplitude, and balance line

2 New Terms

- Period (introduced last lesson)
- Amplitude
- Balance line, balance value
- Sinusoidal function

3 Graphs of Sine and Cosine

- The *balance line* is the centrally located horizontal line about which the sine or cosine graphs oscillate. That is, it is the horizontal line which is halfway in between the maximum and minimum values of the graph. The *balance value* is the y -value of the balance line.
- The *amplitude* is the positive distance between the maximum or minimum value of the sine or cosine and the balance line.
- The *period* is the smallest positive constant k such that $f(x + k) = f(x)$ for all x .

Write at least two equations whose graph corresponds with the one you've drawn. Use the sine for one and the cosine for the other. Introduce the term *sinusoidal*.

If $f(x) = A \sin [B(x - C)] + K$, then

- The amplitude will be $|A|$. This is the vertical stretching or compression from the basic sinusoidal graph.
- The balance value is the average of the maximum and the minimum values. The maximum value of $f(x)$ is $A + K$ and the minimum is $-A + K$ so the balance value is $2K/2 = K$. This is a vertical shift.
- The period will be $\frac{2\pi}{|B|}$. Note that B gives a horizontal stretching or compression which corresponds to lengthening or shortening the period.
- The graph will be shifted C units to the right for $C > 0$. Remember that the sine and the cosine are just shifts of each other. That means if you are given a sinusoidal graph, you can write an equation for the corresponding function in more than one way. For example $\cos(x - \frac{\pi}{2})$ and $\sin x$ have the same graph.

4 References

- §19.2 in *Calculus: An Integrated Approach to Functions and Their Rates of Change*.