# Math Xb Spring 2005
# Tangents, Angles, and Arc Length

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## 1 Goals

- To understand the interpretation of $\tan x$ as the slope of a certain line.
- To see the connections between the graphs of $\sin x$, $\cos x$, and $\tan x$.
- To become comfortable with using radian measure of angles.
- To understand the relationship between angles and arc length.
- To take advantage of circle symmetry when finding trig function values.

## 2 New Terms

- Tangent
- Initial side and terminal side
- Standard position of an angle
- Radian

## 3 The Tangent Function

1. We define the *tangent function* as $\tan x = \frac{\sin x}{\cos x}$.
2. Note that $\tan x$ can be interpreted as the slope of the line connecting the points $(0, 0)$ and the point $P(x) = (\cos x, \sin x)$. There's a discussion of this on page 615 of the textbook.

## 4 Angles and Radian Measure

1. Now that we have drawn the line segment connecting the points $(0, 0)$ and $P(x) = (\cos x, \sin x)$, it makes sense to think of the *angle* this line segment makes with the horizontal axis.
2. While we can measure angles in degrees, there is a more convenient way of measuring angles. An angle of $x$ *radians* is the angle that subtends an arc of length $x$ on the unit circle. The pictures in the book on pages 620 and 621 are helpful here.
3. This allows us to think of, say, $\sin x$ in two different ways.
   (a) We can travel along the unit circle a distance of $|x|$ units from $(1, 0)$ (in the appropriate direction), land on a point, and look at its vertical coordinate.
   (b) We can place an angle measuring $x$ radians in standard position, see where its terminal side intersects the unit circle, and look at the vertical component of that point.

4. For a more general circle of radius $r$, there is also a relationship between the angle $\theta$ in radians and the arc length.

$$\text{arc length} = r\theta$$

5. To convert between radian and degree measure, using the fact that $\pi$ radians $= 180\,\text{deg}$. See p. 621 in the textbook.

# 5 References

- §19.3–19.4 in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*