Math Xb Spring 2005
Right Triangle Trigonometry

February 28, 2005

1 Goals

- To understand the relationship between sine, cosine, and tangent and right triangles.
- To know the sine, cosine, and tangent values of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$ and to be able to use this, along with the symmetry of the right triangle, to find the trigonometric values for angles like $\frac{-5\pi}{6}$
- To "solve" triangles, that is, to determine all angles and sides of a triangle from some given information.
- To define and be able to use the trigonometric functions secant, cosecant, and cotangent.

2 New Terms

- Similar triangles. *Similar triangles* are triangles in which all three angels are of equal measure. This means that the ratio of two corresponding sides is the same in similar triangles. This feature is what makes right triangle trigonometry work! See page 628 in the textbook for more details.
- Secant
- Cosecant
- Cotangent
- Angle of elevation and angle of depression: *Angle of elevation* refers to the angle from the horizontal up to an object (such as the top of a tree). *Angle of depression* refers to the angel from the horizontal down to an object (such as the bottom of a hole).
- Complementary angles: Two angles are *complementary* if their sum is $\pi/2$ (or 90° if you are measuring in degrees. In a right triangle, the two acute angles are always complementary.

3 Right-Triangle Trigonometry

1. Note that we have defined sine, cosine, and tangent as functions of x, where we can think of x as a directed distance around the unit circle or as an angle measured in radians.
2. Today we make the connection between the trig functions and right triangles.
3. Draw a right triangle with an acute angle θ and label the sides *opposite*, *adjacent*, and *hypotenuse*.
4. Then "shrink" this triangle so that it fits in the unit circle with θ in standard position. The hypotenuse of this new triangle should have length 1. In similar triangles the ratio of any two corresponding sides is the same. We can then make the following observations.
   (a) $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
   (b) $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
   (c) $\tan\theta = \frac{\text{opp}}{\text{adj}}$

4 Cosecant, Secant, and Cotangent

1. The reciprocals of the sine, cosine, and tangent functions are useful enough that they have their own names.

   (a) $\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$

   (b) $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$

   (c) $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} = \frac{\text{adj}}{\text{opp}}$

5 The $45°$-$45°$-$90°$ Triangle

1. We can use an isosceles right triangle to find the sine and cosine values of $\frac{\pi}{4}$.

2. We can use the symmetry of the unit circle to find the sine, cosine, and other trig functions for angles of $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, $\frac{7\pi}{4}$, and many other angles.

6 The $30°$-$60°$-$90°$ Triangle

1. We can use an equilateral triangle to find the sine and cosine values of $\frac{\pi}{3}$ and $\frac{\pi}{6}$ (these are complementary angles).

2. We can use the symmetry of the unit circle to find the sine, cosine, and other trig values for angles of $\frac{2\pi}{3}$, $\frac{5\pi}{6}$, and many other angles.

7 References

- §20.1–20.2 in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*