Math Xb Spring 2005
Inverse Trigonometric Functions

March 2, 2005

1 Goals

- To understand the inverse trig functions $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ and their domains and ranges.
- To simplify expressions involving inverse trig functions by using triangles

2 New Terms

- Arcsine: the inverse of the sine function, also written as $\sin^{-1}$.
- Arccosine: the inverse of the cosine function, also written as $\cos^{-1}$.
- Arctangent: the inverse of the tangent function, also written as $\tan^{-1}$.

3 Inverse Trig Functions

Briefly review of what we know about inverse functions.

1. The function f has an inverse if and only if it is one-to-one.
2. If f is one-to-one, then $f^{-1}(y) = x$ if and only if $f(x) = y$.
3. The graph of f^{-1} can be obtained by reflecting the graph of f across the line $y = x$.

The sine, cosine, and tangent functions are *not* one-to-one. Thus we need to restrict their domains if we want to define inverse functions. For the sine and the tangent we restrict to the domain from $-\pi/2$ to $\pi/2$ (inclusive for the sine, but not inclusive for the tangent since the tangent isn't defined at the endpoints). For the cosine we restrict to the domain $[0, \pi]$.

We define the inverse trig functions for sine, cosine, and tangent as follows.

- The arcsine, denoted $\arcsin x$ or $\sin^{-1} x$ is the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sine is x. The domain of $\sin^{-1} x$ is $[-1, 1]$. The range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
- The arccosine, denoted $\arccos x$ or $\cos^{-1} x$ is the angle between 0 and π whose cosine is x. The domain of $\cos^{-1} x$ is $[-1, 1]$. The range is $[0, \pi]$.
- The arctangent, denoted $\arctan x$ or $\tan^{-1} x$ is the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is x. The domain of $\tan^{-1} x$ is $(-\infty, \infty)$. The range is $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Graphs of the inverse trig functions can be found on p. 647 of the textbook.

4 Examples

5 References

- §20.3 in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*