Math Xb Spring 2005
Solving Trigonometric Equations

March 4, 2005

1 Goals

- To solve equations involving trig functions on both restricted and unrestricted domains

2 Solving Equations

These problems constitute Problem #1 on the worksheet.

1. $\sin x = \frac{1}{2}$ for $x \in [0, 2\pi]$. Be careful about the domain here, just using $\sin^{-1}$ won't work. We know that $\sin x = \frac{1}{2}$ at $x = \frac{\pi}{6}$ and by symmetry it also happens at $x = \frac{5\pi}{6}$. Between π and 2π the sine is negative, so none of those values will work.

2. $\sin x = \frac{1}{2}$ for any real x. We've already solved this problem on a restricted domain. We can get a general solution by remembering that the sine has a period of 2π, so every time we add a multiple of 2π to $x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$, we'll still get $\sin x = \frac{1}{2}$. So the solution is $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{5\pi}{6} + 2k\pi$, for k any integer.

3. $\cos 2x = 0$ for $x \in [-2\pi, 2\pi]$. We know that $\cos u = 0$ for $u = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$. So we solve for x where $2x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ to get $x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$.

4. To solve an equation of the form
$$5u = u^2$$
you cannot divide both sides by u! This is because u could be equal to zero. Instead, bring all of the terms to one side and factor. When you have a quadratic that you can't factor, use the quadratic formula.

5. $\sin\theta \tan\theta = \sin\theta$. Collect all the terms to one side and factor.
$$\sin\theta \tan\theta = \sin\theta$$
$$\sin\theta \tan\theta - \sin\theta = 0$$
$$\sin\theta(\tan\theta - 1) = 0$$

Thus either $\sin\theta = 0$ or $\tan\theta - 1 = 0$. If $\sin\theta = 0$, then θ is a multiple of π so $\theta = k\pi$, where k is any integer. If $\tan\theta - 1 = 0$, then $\tan\theta = 1$, so $\theta = \frac{\pi}{4} + 2k\pi$ or $\theta = \frac{5\pi}{4} + 2k\pi$ for k any integer.

6. $2\sin^2 t - \cos t = 1$ for $x \in [0, 2\pi]$. For this problem, the trick is to convert $\sin^2 t$ into an expression in the cosine using $\sin^2 t + \cos^2 t = 1$. Then collect the terms on one side and factor.

3 References

- §20.4 in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*