Math Xb Spring 2005
Laws of Sines, Cosines, and Trig Identities

March 7, 2005

1 Goals

- To be able to apply the Law of Cosines and the Law of Sines.
- To realize that $\sin(A + B) \neq \sin A + \sin B$.
- To understand how many of the trigonometric identities can be derived from just a few identities.
- Be able to use the addition formulas and other identities to simplify trig expressions

2 New Terms

- Oblique triangle
- Law of Cosines
- Law of Sines
- Trigonometric Identity

3 Law of Sines

We can "solve" a right triangle using the three basic trigonometric functions. "Solving" *oblique* triangles (those that are not right triangles) is harder, but we do have some tools for doing so.

1. Denote the angles of a given triangle by A, B, and C. Denote the lengths of the sides opposite angles A, B, and C by a, b, and c, respectively.
2. The area of a triangle is given by $\frac{1}{2}\text{base} \cdot \text{height}$. Suppose that we know the lengths of two sides a and c of a triangle and the angle C between them. If y is the height of the triangle and b is the base, then $y = a \sin C$ (see the pictures at the bottom of page 663 and the discussion on 663—664). Thus the area of the triangle is
$$\text{Area} = \tfrac{1}{2}\text{base} \cdot \text{height} = \frac{1}{2} ab \sin C.$$
3. The preceding formula works if we pick any two sides and the angle between them. So we can pick a, b, and angle C as above. If we pick a, c, and angle B we get that the area is $\frac{1}{2} ac \sin B$, and if we pick b, c, and angle A we get $\frac{1}{2} bc \sin A$. But the area must be the same in all three cases, so
$$\frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B = \frac{1}{2} bc \sin A$$
Simplifying gives the *Law of Sines*
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$
4. The Law of Sines can be used to solve the introductory problem.

4 Law of Cosines

Another tool we have to "solve" oblique triangles is the *Law of Cosines*

1. The *Law of Cosines* states that
$$c^2 = a^2 + b^2 - 2ab\cos C.$$
Explain how we can think of this as the Pythagorean Theorem with an adjustment term. You need not prove the Law of Cosines.

2. The Law of Cosines can be used to solve the second problem on the worksheet.

5 Trig Identities

A *Trigonometric Identity* is a trigonometric equation that is true for *all* values of the inputs. Note that the Law of Cosines and the Law of Sines are *not* trigonometric identities since they are only true for some values of inputs (those where a, b, and c make a triangle). Here are some trigonometric identities we already know which are true for all values of x for which the expressions make sense.

1. $\tan x = \frac{\sin x}{\cos x}$
2. $\sec x = \frac{1}{\cos x}$
3. $\csc x = \frac{1}{\sin x}$
4. $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$
5. $\sin^2 x + \cos^2 x = 1$
6. $\sin(-x) = -\sin x$
7. $\cos(-x) = \cos x$

Two other trig identities which are very important are the *angle-addition formulas*.

$$\sin(x + y) = \sin x \cos y + \sin y \cos x$$
$$\cos(x + y) = \cos x \cos y - \sin x \cos y$$

We can use these trig identities to derive other useful identities. You can find several examples of this in section 20.6 of your textbook. The next page contains trigonometric identities which are are expected to know. These identities can be derived from the identities listed above, plus the pythagorean theorem.

6 References

- §20.5–20.6 in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*

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Handout: Trigonometric Identities
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You will be responsible for knowing the following trigonometric identities.

Definitions

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Symmetry Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

Angle-Addition/Subtraction Identities

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

Power-Reducing Identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$