Math Xb Spring 2005
Derivatives of Trig Functions

March 9, 2005

1   The Derivative of Sine

1. A tangent line analysis of $y = \sin x$ leads one to supect that $y' = \cos x$.

2. We can prove that t $\frac{d}{dx}(\sin x) = \cos x$. The proof in the book on page 688 (and page 689 to 691 discusses the computation of two limits involved in finding the derivative).

2   Derivative of the Cosine

Recall that
$$\cos x = \sin\left(x + \frac{\pi}{2}\right).$$

Thus we can find the derivative of the cosine as follows

$$\begin{aligned}
\frac{d}{dx}\cos x &= \frac{d}{dx}\sin\left(x + \frac{\pi}{2}\right) \\
&= \cos\left(x + \frac{\pi}{2}\right) \cdot \frac{d}{dx}\left(x + \frac{\pi}{2}\right) \\
&= \cos\left(x + \frac{\pi}{2}\right) \cdot 1 \\
&= \cos\left(x + \frac{\pi}{2}\right) \\
&= -\sin x
\end{aligned}$$

3   References

- §21.1–21.2 in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*