Math Xb Spring 2005
L'Hôpital's Rule Day Two

March 23, 2005

1   Goals

- To recognize the indeterminate forms $0 \cdot \infty$, 1^{∞}, ∞^0, and 0^0.

- To evaluate the above forms using L'Hôpital's Rule.

2   The Indeterminate Form $0 \cdot \infty$

1. We can already evaluate the following limits

$$\lim_{x \to \infty} \frac{1}{x^2} x = 0 \tag{1}$$

$$\lim_{x \to \infty} \frac{1}{x} x^2 = \infty \tag{2}$$

$$\lim_{x \to \infty} \frac{10}{x} x = 10 \tag{3}$$

2. Note that each of these limits is indeterminate of the form $0 \cdot \infty$. The form is indeterminate since the limit could turn out be any constant or to be infinite.

3. We can evaluate limits of indeterminate form $0 \cdot \infty$ using L'Hôpital's Rule, but we first have to convert the form to type $0/0$ or ∞/∞.

   (a) $\lim\limits_{x \to \infty} \dfrac{1}{e^x} \ln x$. This is of the form $0 \cdot \infty$. Note that $e^{-x} = \frac{1}{e^x}$, so

$$
\begin{aligned}
\lim_{x \to \infty} e^{-x} \ln x &= \lim_{x \to \infty} \frac{\ln x}{e^x} \quad \text{Form } \infty/\infty \\
&= \lim_{x \to \infty} \frac{\frac{d}{dx} \ln x}{\frac{d}{dx} e^x} \quad \text{L'Hôpital's rule} \\
&= \lim_{x \to \infty} \frac{\frac{1}{x}}{e^x} \\
&= 0
\end{aligned}
$$

   (b) $\lim\limits_{x \to 1^+} \left[(x-1) \tan\left(\dfrac{\pi x}{2}\right) \right]$. This is of the form $0 \cdot \infty$. We can rewrite

$$(x-1) \tan\left(\frac{\pi x}{2}\right) = \frac{x - 1}{\left(\frac{1}{\tan\left(\frac{\pi x}{2}\right)} \right)}$$

   so that the form of the limit becomes $0/0$. We can then apply L'Hôpital's rule to find the limit.

3   Indeterminate Forms 1^{∞}, ∞^0, and 0^0

1. The indeterminate forms 1^{∞}, ∞^0, and 0^0 are called the exponential indeterminate forms.

2. To calculate limits of these forms using L'Hôpital's Rule, we first have to take the natural log. The procedure is discussed on p. 1117 of the textbook.

4 References

- Appendix F in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*