

# Math Xb Spring 2005

## The Area Function

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### 1 Goals

- To find the area function  ${}_aA_f$  for a simple function  $f$ .
- To interpret the area function as a net change function
- To know why two area functions  ${}_aA_f$  and  ${}_bA_f$  differ by a constant

### 2 New Terms

- Area function

### 3 Area Functions

1. Suppose a car's velocity is given by the function  $f(t) = 3t + 1$  on the time interval  $[0, 10]$ . We know that the net change in position of the car on that time interval is  $\int_0^{10} f(t) dt$ . We also know that the definite integral  $\int_0^{10} f(t) dt$  represents the area under the graph of  $y = 3t + 1$  from  $t = 0$  to  $t = 10$ .
2. What about  $\int_0^x f(t) dt$ , where we let  $x$  vary? Interpret  $\int_0^x f(t) dt$  in terms of net change and in terms of area under the curve. This determines a function of  $x$  since the area under the curve is completely determined by the value of  $x$ .
3. *Definition:* Suppose  $f$  is a continuous function and  $a$  is a number in the domain of  $f$ . Define the function  ${}_aA_f$  by the following rule.

$${}_aA_f(x) = \int_a^x f(t) dt$$

Since  ${}_aA_f(x)$  gives the signed area between the graph of  $f$  and the horizontal axis between  $t = a$  and  $t = x$ , we call this function the *area function*.

4. If  $a$  and  $b$  are two different constants, then  ${}_aA_f(x)$  and  ${}_bA_f(x)$  differ only by a constant. The reason for this is discussed on p. 745 of the textbook.

### 4 References

- §23.1 in *Calculus: An Integrated Approach to Functions and Their Rates of Change*.