Math Xb Spring 2005
Modeling with Differential Equations

April 29, 2005

1 Goals

- To interpret a given differential equation in the context of a particular application.
- To write a differential equation which models a given situation.

2 A few notes on interpretation

1. In a differential equation $dP/dt = \cdots$, if the term kP appears on the right-hand side somewhere, where k is a constant, then this means the rate of change is, to some extent, proportional to P. Population growth works like this because the number of babies born depends on the number of creatures out there having babies. Bank accounts work like this because the interest an account makes is a percentage of the money already in the account.
2. If a constant term appears on the right hand side, that represents a flat, constant amount that is added or subtracted every time period. For example, you might put \$100 into a savings account each month, which would be a "+100" on the right hand side of a differential equation $dS/dt = \cdots$ modeling your savings account.
3. If a *function* of t appears on the right hand side, this represents some part of the rate that is changing just because of the passage of time. For example, if each year because of inflation we have to take 50 *more* out of our bank account than the year before in order to cover our expenses, this would give a term "$-50t$" on the right-hand side of an equation $dS/dt = \cdots$ modeling our bank account.

3 Note on Percent Growth

It is tempting, if we want to model a population growing at a rate of 3% per year to write the differential equation

$$\frac{dP}{dt} = .03P.$$

The trouble is that this ignores the fact that we are using continuous equations to model our population growth. In some sense, we are continuously compounding our population growth. We actually need to model our growth as

$$P(t) = P_0 e^{kt}$$

So that if we want a 3% annual growth rate that means we want $P(1) = 1.03P_0$ so that

$$\begin{aligned} 1.03P_0 &= P_0 e^k \quad \text{Since } t = 1 \\ 1.03 &= e^k \\ \ln 1.03 &= k \end{aligned}$$

4 References

- §31.1 in *Calculus: An Integrated Approach to Functions and Their Rates of Change.*