

Related Rates Problems

- A balloon is rising at a constant speed of 5 ft/sec. A boy is cycling along a straight road at a speed of 15 ft/sec. When the boy passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 seconds later?
- Oil is leaking from an ocean tanker at the rate of 5000 liters per minute. The leakage results in a circular oil slick. The depth of the oil varies linearly from a maximum of 5 cm at the point of leakage to minimum of 0.5 cm at the outside edge of the slick. How fast is the radius of the slick increasing 4 hours after the tanker started leaking? [*Hints:* The volume of a cone is $V = (1/3)\pi r^2 h$. 1 liter = 1000 cm³. Be careful of units.]

Implicit Differentiation

Use implicit differentiation to find an equation of the tangent line to the ellipse

$$5x^2 - 6xy + 5y^2 = 16$$

at the point $(1, -1)$.

Optimization Problems

A Norman window has the shape of a rectangle surmounted by a semicircle. Thus, the diameter of the semicircle is equal to the width of the rectangle. If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest amount of light is admitted.

Strategies for Optimization Problems

1. Draw the picture.
2. Name the variables.
3. Write the function that is to be optimized.
4. Relate the variables.
5. Write the function that is to be optimized as a function of a single variable.
6. Differentiate and find the critical points.
7. Evaluate the function at the critical points and endpoints or use the first or second derivative test.
8. Write your solution.

Logarithms and Differentiation

Find the derivatives of each of the following.

- $y = \ln \left(\frac{x\sqrt{1+x^2}}{(x^2-3x+7)^9} \right)$

- $y = \frac{x\sqrt{1+x^2}}{(x^2-3x+7)^9}$

- $y = x^{\ln \sqrt{x}}$

L'Hôpital's Rule and Indeterminate Forms

Find the following limits.

- $\lim_{x \rightarrow \infty} \frac{6x^2 - 3x - 5}{2x^2 - 3x + 5}$

- $\lim_{x \rightarrow 2} \frac{x^2 - 4}{(x - 2)(x + 3)}$

- $\lim_{x \rightarrow 2} \frac{x^2 - 4}{f(x)}$, given the graph of f

- $\lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^x$