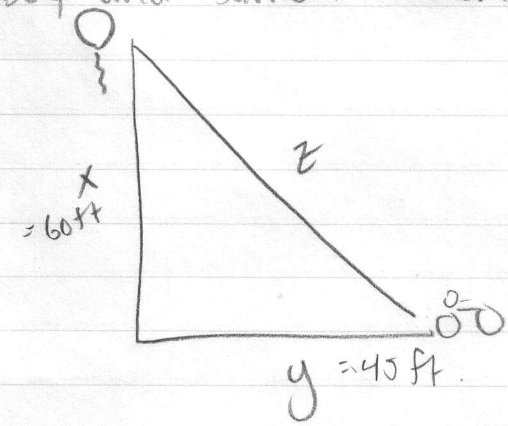
first
1/3 of course
Math Review Session 5/12/05 (Th)

Related Rates Problems, optimization, L'Hôpital's Rule

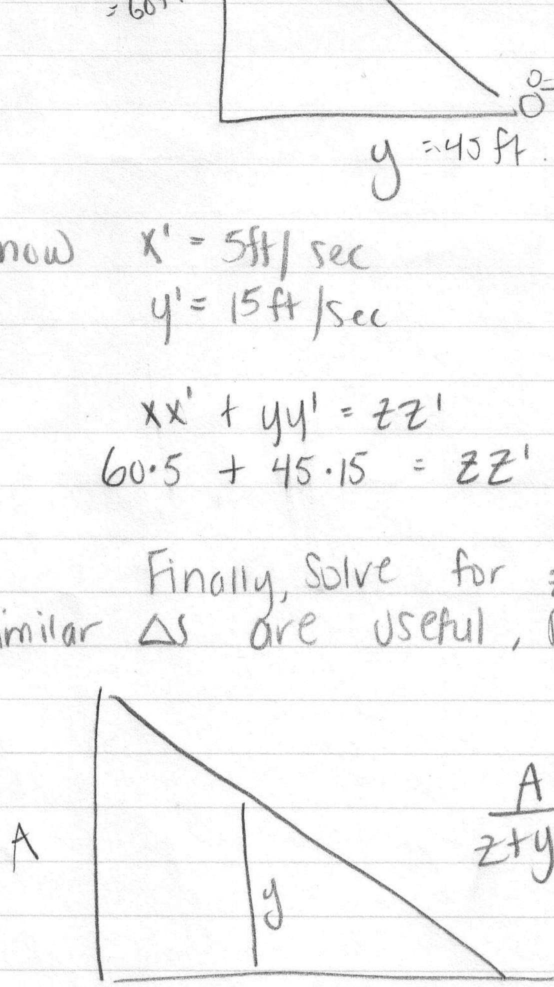
5ft/sec ⇒ balloon is rising
boy cycling along rd. at 15 ft/sec
When boy passes under balloon it is 45 ft above him. How fast is the distance between boy and balloon increasing 3 sec later?



$x^2 + y^2 = z^2$
$2x\,x' + 2y\,y' = 2z\,z'$

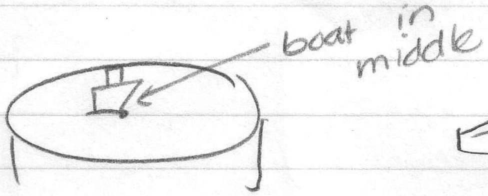
Know $x' = 5$ ft/sec
$y' = 15$ ft/sec

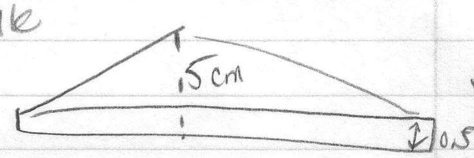
$xx' + yy' = zz'$
$60 \cdot 5 + 45 \cdot 15 = zz'$; Now we need z (use P.T.)
$z = \sqrt{60^2 + 45^2}$
Finally, Solve for z'.
Similar △s are useful, P.T. invoked OFTEN

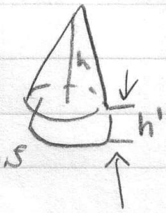


$\frac{A}{z+y} = \frac{b}{x}$

Oil leaking at 5000 liters / min
Leakage results in a circular oil slick







Depth of oil varies linearly from a maximum of 5 cm at the pt of leakage to minimum of .5 cm at outside edge of the slick.
How fast is the radius of the slick increasing 4 hrs after the tanker started leaking? ($V = 1/3 \pi r^2 h$ and 1 liter = 1000 cm^3)

$$V = \frac{1}{3}\pi R^2 h + \pi R^2 h'$$
$$= \frac{1}{3}\pi R^2 (4.5) + \pi R^2 (0.5)$$

$$V = 2\pi R^2$$

radius & volume are functions of time

$$5000 \times 10^3 = \frac{dV}{dt} = 4\pi R \frac{dR}{dt}$$

$$5000 \times 10^3 \cdot 60 \cdot 4 = 2\pi R^2$$

Now find $\frac{dR}{dt}$

do the metric conversion

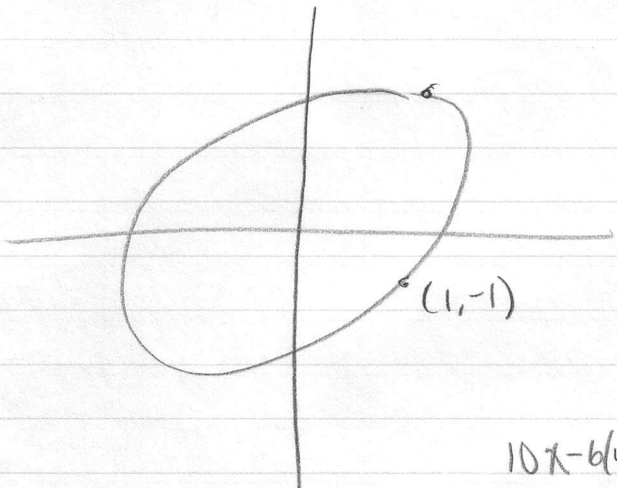
Break down into pieces; look for volumes.

Look for volumes, Similar Δs and the Pythagorean Theorem

Implicit Differentiation - 2nd topic

Find an equation of the tangent line to the ellipse

$5x^2 - 6xy + 5y^2 = 16$ at the point $(1,-1)$



Assume y is a function of x.

$10x - 6(y + xy') + 10yy' = 0$

$10 - 6(-1 + y') - 10y' = 0$

$16 - 6y' - 10y' = 0$

$16y' = 16 \Rightarrow y' = 1$

Unfinished!

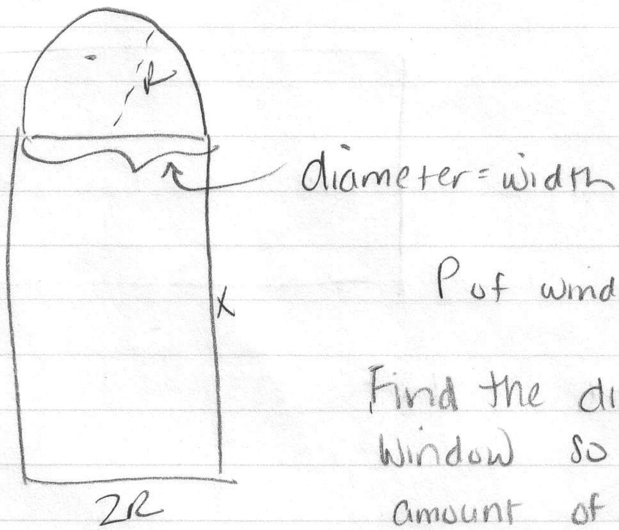
$y - (-1) = 1(x - 1)$

$y + 1 = x - 1$

$\boxed{y = x - 2}$ ⟸ final answer

Remember Product Rule & Chain Rule

☆ Optimization Norman window



P of window = 30 ft.

Find the dimensions of the window so the greatest amount of light is admitted.

A = AREA
P = 30 ft.

$A = 2xR + \frac{\pi}{2}R^2$

Now, relate the variables ...

$30 = \pi R + 2x + 2R$

$2x = 30 - \pi R - 2R$

$x = 15 - \frac{\pi}{2}R - R$

Rewrite top formula

$A = 2(15 - \frac{\pi}{2}R - R)R + \frac{\pi}{2}R^2$

$A = 30R - \pi R^2 - 2R^2 + \frac{\pi}{2}R^2$

$A = 30R - \frac{\pi}{2}R^2 - 2R^2$

Differentiate, find critical pts.
Use first and second deriv test
Test endpts to see if you have a local max.

Can't be .

Logarithms and Differentiation

① $y = \ln\left(\frac{x\sqrt{1+x^2}}{(x^2-3x+7)^9}\right)$

$y = \ln x + \frac{1}{2}\ln(1+x^2) - 9\ln(x^2-3x+7)$

$y' = \frac{1}{x} + \frac{1}{2}\,\frac{1}{1+x^2}\,2x - 9\,\frac{1}{x^2-3x+7}(2x-3)$

② $y = \frac{x\sqrt{1+x^2}}{(x^2-3x+7)^9}$

$\ln y = \ln\left(\frac{x\sqrt{1+x^2}}{(x^2-3x+7)^9}\right)$

$\frac{y'}{y} =$

; then multiply both sides by y.

③ $y = x^{\ln\sqrt{x}}$

$\ln y = \ln x^{\ln\sqrt{x}}$

$\ln y = \frac{1}{2}\ln x \ln x$

$\ln y = \frac{1}{2}(\ln x)^2$

$\frac{y'}{y} = \ln x \frac{1}{x}$

$$\boxed{y' = \frac{\ln x}{x} \cdot x^{\ln\sqrt{x}}}$$

$\frac{1}{x} \Rightarrow -\frac{1}{x^2}$

L'Hôpital's Rule & Indeterminate Forms

- $\lim\limits_{x \to \infty} \dfrac{6x^2-3x-5}{2x^2-3x+5} = 3$ $\left(\frac{\infty}{\infty}\right)$

$= \lim\limits_{x \to \infty} \dfrac{12x-3}{4x-3}$ $\left(\frac{\infty}{\infty}\right)$

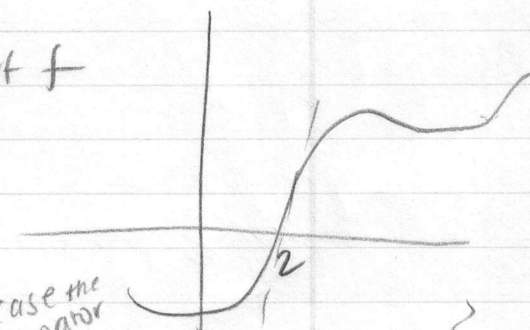
$= \lim\limits_{x \to \infty} \dfrac{12}{4} = 3$

- $\lim\limits_{x \to 2} \dfrac{x^2-4}{(x-2)(x+3)} = \left(\dfrac{0}{0}\right)$

must be multiplied out

$\lim\limits_{x \to 2} \dfrac{2x}{2x+1} = \dfrac{4}{5}$

- $\lim\limits_{x \to 2} \dfrac{x^2-4}{f(x)}$, given the graph of f $\left(\frac{0}{0}\right)$



$\lim\limits_{x \to 2} \dfrac{2x}{f'(x)}$ ← slope of about 2 $= 2$

in case the denominator is a definite integral, find the derivative of the integral

$\dfrac{x^2-4}{\int_0^x f(t)\,dt}$

★ - $\lim\limits_{x \to \infty} \left(1+\frac{5}{x}\right)^x = \left(1^\infty\right)$

$\ln y = \ln\left(1+\frac{5}{x}\right)^x$

$\lim\limits_{x \to \infty} = x \ln\left(1+\frac{5}{x}\right)$ $(\infty \cdot 0)$

$\lim\limits_{x \to \infty} \dfrac{\ln\left(1+\frac{5}{x}\right)}{1/x}$ top and bottom go to zero $= \left(\dfrac{0}{0}\right)$

$\lim\limits_{x \to \infty} \dfrac{\frac{1}{1+\frac{5}{x}} \cdot 5 \cdot \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}}$ as $x \to \infty$ $= 5$ $\ln y = 5$ $\lim = e^5$